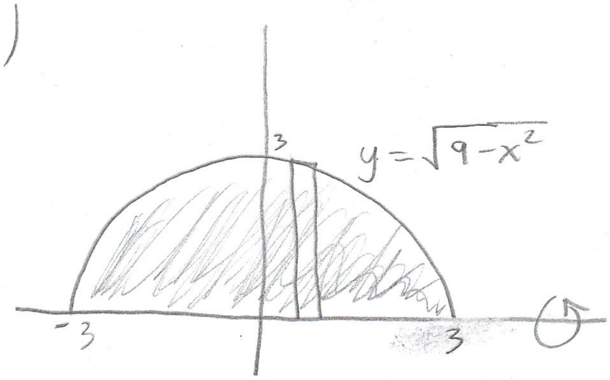
Volume - Solids of Revolution WS1

1)



cross section: disc

set up:

$$V=\pi\int_{-3}^{3}\left(\sqrt{9-x^2}\right)^2dx$$

$$V=\pi\int_{-3}^{3}9-x^2\,dx$$

$$V=\pi\left(9x-\frac{x^3}{3}\right|_{-3}^{3}=\pi(27-9)-(-27+9)$$

$$V=\pi(18)-(-18)=\boxed{36\pi}$$

2) $y=\sec x$, x-axis from $x=-\frac{\pi}{4}$ to $x=\frac{\pi}{4}$ about x-axis

setup: $V=\pi\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}(\sec x)^2dx$

$$V=\pi\left(\tan x\right|_{-\frac{\pi}{4}}^{\frac{\pi}{4}}=\pi\left(\tan\frac{\pi}{4}-\tan\left(-\frac{\pi}{4}\right)\right)=\pi(1-(-1))$$

$$=\boxed{2\pi}$$

3) cross sections are squares, $V=\overset{\text{integral}}{\int}$ area of cross section

$A_{square}=s^2$

If a side of the square lies on the base of the object Then it must be bounded by the function $y=\sqrt{16-x^2}$.

$y=\sqrt{16-x^2}$ is the top half of a semi-circle with a radius of 4.

$\therefore$ $s=\sqrt{16-x^2}$

set up: $V = \int \left(\sqrt{16-x^2}\right)^2 dx$

limits of integration can be found by finding when the function $y = \sqrt{16-x^2}$ and the x-axis $(y=0)$ intersect

$0 = \sqrt{16-x^2}$

$0 = 16-x^2$

$x^2 = 16$

$x = \pm 4$

set up: $V = \int_{-4}^{4} 16 - x^2\, dx$

$V = \left(16x - \frac{x^3}{3}\right|_{-4}^{4} = \left(16(4) - \frac{4^3}{3}\right) - \left(16(-4) - \frac{(-4)^3}{3}\right)$

$V = \left(64 - \frac{64}{3}\right) - \left(-64 + \frac{64}{3}\right) = \frac{128}{3} - \left(-\frac{128}{3}\right) = \boxed{\frac{256}{3}}$

4) $x = 1 - y^2$ and y-axis $(x=0)$ intersect

$0 = 1 - y^2$

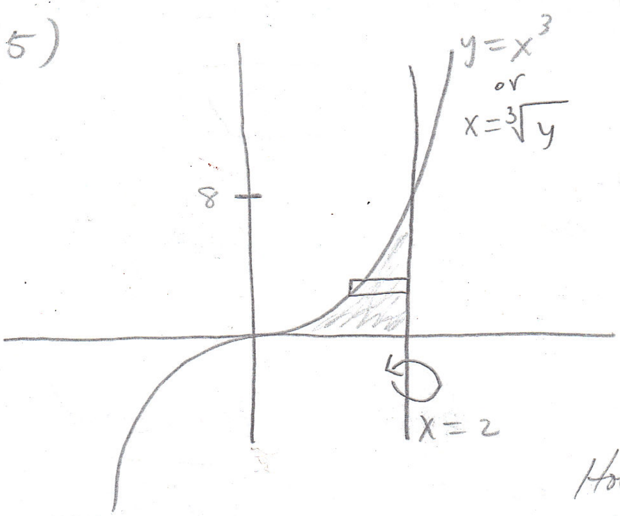
$y^2 = 1$

$y = \pm 1 \leftarrow$ limits of integration

set up: $V = \pi \int_{-1}^{1} (1-y^2)^2 dy = \pi \int_{-1}^{1} 1 - 2y^2 + y^4\, dy = \pi \left(y - \frac{2y^3}{3} + \frac{y^5}{5}\right|_{-1}^{1}$

$\Rightarrow V = \pi \left(1 - \frac{2}{3} + \frac{1}{5}\right) - \left(-1 + \frac{2}{3} - \frac{1}{5}\right) = \pi \left(\frac{8}{15}\right) - \left(-\frac{8}{15}\right) = \boxed{\frac{16}{15}\pi}$

5)



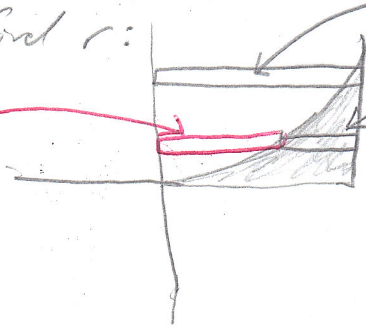
Cross section: disc

$V = \pi \int r^2\, dy$

r is the length of the rectangle

b/c axis of revolution is a vertical line like the y-axis

How to find r:



length is $\sqrt[3]{y}$

length is 2

length is $2 - \sqrt[3]{y}$

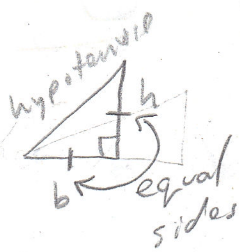
or Right - Left

set up: $V = \pi \int_0^8 (2 - \sqrt[3]{y})^2 \, dy = \pi \int_0^8 4 - 4\sqrt[3]{y} + \sqrt[3]{y^2} \, dy$

$\Rightarrow V = \pi \int_0^8 4 - 4y^{1/3} + y^{2/3} \, dy = \pi \left(4y - 3y^{4/3} + \frac{3y^{5/3}}{5} \right) \Big|_0^8$

$\Rightarrow V = \pi \left(4(8) - 3(8)^{4/3} + \frac{3(8)^{5/3}}{5} \right) = \pi \left(32 - 48 + \frac{96}{5} \right) = \pi \left(-16 + \frac{96}{5} \right)$

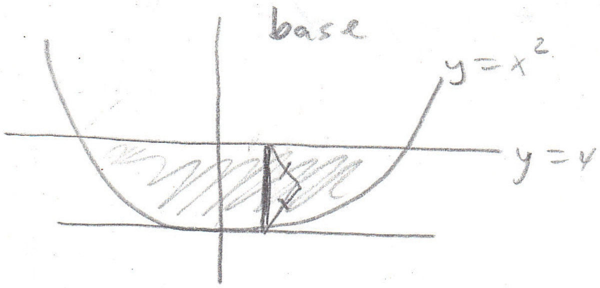
$\Rightarrow V = \pi \left(\frac{-80}{5} + \frac{96}{5} \right) = \boxed{\frac{16\pi}{5}}$

6) cross sections are isosceles right Δ's, 

, $A = \frac{1}{2}bh$

$A = \frac{1}{2}b(b)$

$A = \frac{1}{2}b^2$



hypotenuse lies on the base, $\therefore$

hypotenuse = top − bottom

$= 4 - x^2$

intersection:

$x^2 = 4$

$x = \pm 2$

according to pythagoream then

$a^2 + b^2 = c^2$

$b^2 + b^2 = (4 - x^2)^2$

$2b^2 = (4 - x^2)^2$

$b^2 = \frac{(4 - x^2)^2}{2}$

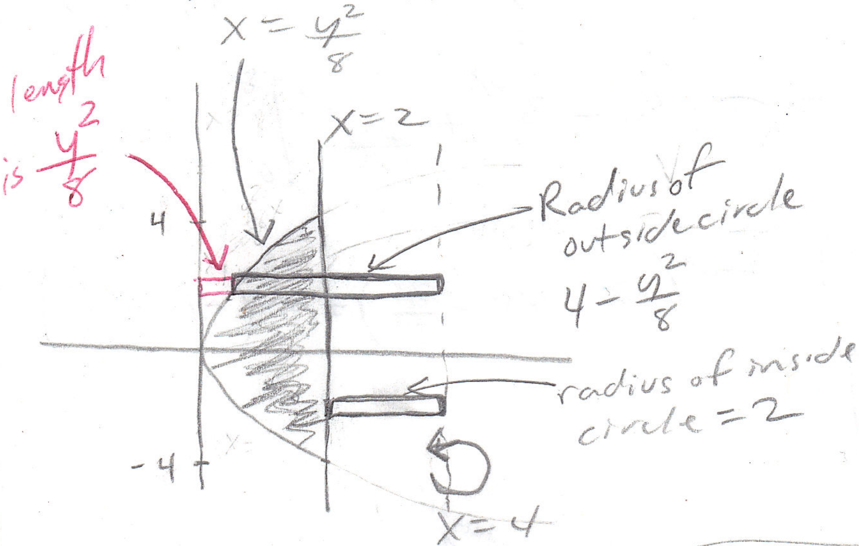
set up: $V = \int_{-2}^{2} \frac{1}{2} \left(\frac{(4 - x^2)^2}{2} \right) dx = \frac{1}{4} \int_{-2}^{2} (4 - x^2)^2 \, dx = \frac{1}{4} \int_{-2}^{2} 16 - 8x^2 + x^4 \, dx$

$\Rightarrow V = \frac{1}{4} \left(16x - \frac{8x^3}{3} + \frac{x^5}{5} \right) \Big|_{-2}^{2} = \frac{1}{4} \left(16(2) - \frac{8(2)^3}{3} + \frac{2^5}{5} \right) - \left(16(-2) - \frac{8(-2)^3}{3} + \frac{(-2)^5}{5} \right)$

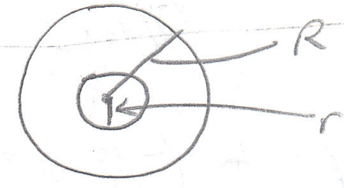
$\Rightarrow V = \frac{1}{4} \left[\left(32 - \frac{64}{3} + \frac{32}{5} \right) - \left(-32 + \frac{64}{3} - \frac{32}{5} \right) \right] = \frac{1}{4} \left[\left(\frac{256}{15} \right) - \left(-\frac{256}{15} \right) \right] = \frac{1}{4} \left(\frac{512}{15} \right)$

$\Rightarrow V = \boxed{\frac{128}{15}}$

7) $y^2 = 8x$, $x = 2$ revolved around $x = 4$



Cross section is a washer



Set up: $\boxed{V = \pi \int_{-4}^{4} \left(4 - \frac{y^2}{8}\right)^2 - (2)^2 \, dy}$