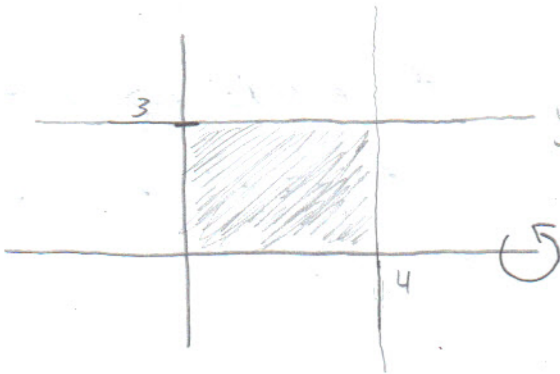
Volume - solids of revolution

1] Take a rectangle generated by the region bounded by $y = 3$, $x = 4$, y-axis and the x-axis.



we revolve the area around the x-axis. What shape do we get? A: cylinder

we know the formula for the volume of a cylinder

$V = \pi r^2 h$

what is r? what is h?

$V = \pi (3)^2 (4) = 36\pi$

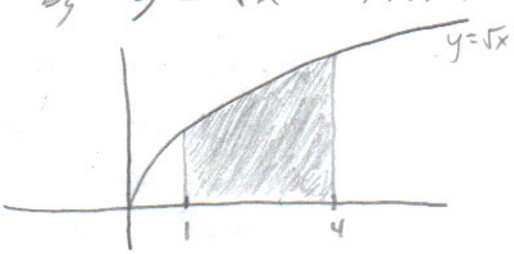
Now, with calculus: $V = \int_a^b$ Area of a cross section $= \int_a^b \pi r^2$

a "slice" of the object

$$V = \int_{x=0}^{x=4} \pi r^2 \, dx = \pi \int_0^4 3^2 \, dx = \pi \int_0^4 9 \, dx$$

$$= \pi \, 9x \Big|_0^4 = \pi (36 - 0) = 36\pi$$

2] Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ from $x = 1$ to $x = 4$ about the x-axis



$\pi \int_a^b r^2 =$

what is r? A: $\sqrt{x}$

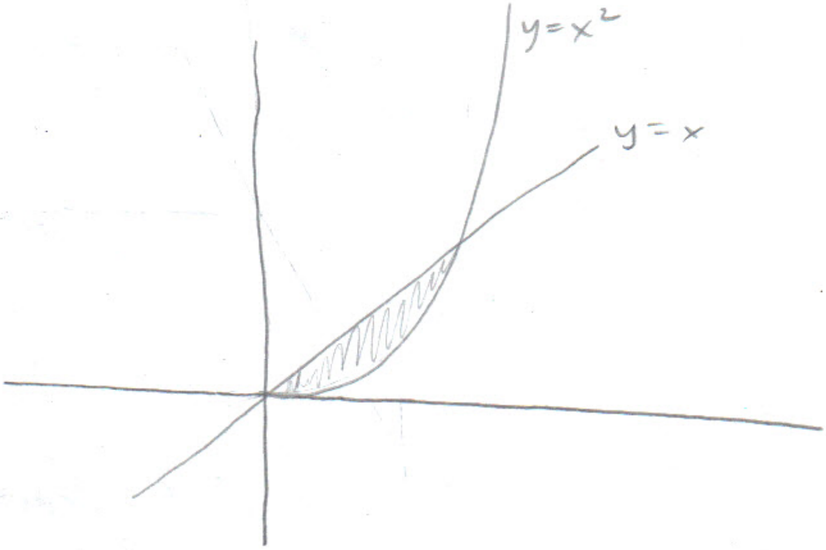
curves will be your radii

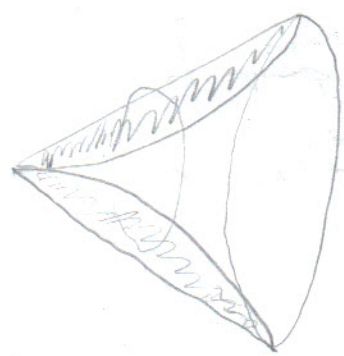
$$\pi \int_a^b r^2 = \pi \int_1^4 (\sqrt{x})^2 dx = \pi \int_1^4 x \, dx = \pi \left(\frac{x^2}{2} \right) \Big|_1^4$$

$$= \pi \left(\frac{16}{2} - \frac{1}{2} \right) = \frac{15\pi}{2}$$

Trickier example:

3] Find the volume of the solid that results when the region between the curves $y = x$ and $y = x^2$ from $x = 0$ to $x = 1$ is revolved about the x-axis.





$V = \pi \int_a^b r^2$ but what is r?

A: furthest curve from x-axis minus closest curve to x-axis (like top - bottom except for under x-axis)

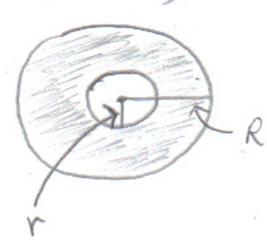
$$V = \pi \int_0^1 (x)^2 - (x^2)^2 dx$$

$$V = \pi \int_0^1 x^2 - x^4 dx$$

$$V = \pi \left(\frac{x^3}{3} - \frac{x^5}{5} \right) \Big|_0^1$$

$$V = \pi \left(\frac{1}{3} - \frac{1}{5} \right) = \frac{2\pi}{15}$$

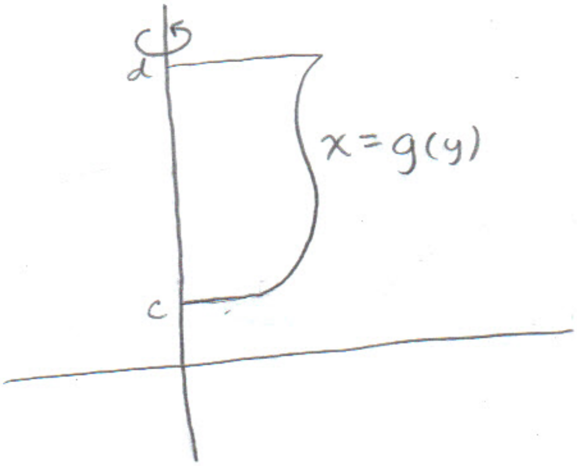
Cross section:



Area between curves is Area of outside - Area of inside

$$\pi R^2 - \pi r^2 = \pi (R^2 - r^2)$$

Volumes revolved around the y-axis



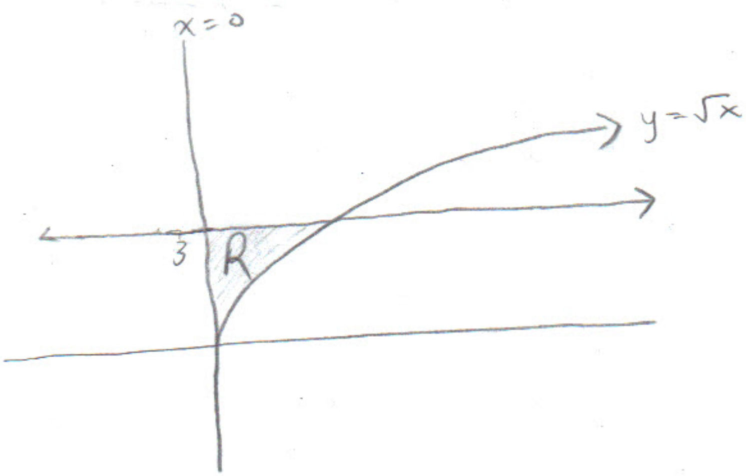
$$V = \int_c^d \pi [g(y)]^2 \, dy$$

Find the volume of the solid generated when the region enclosed by $y = \sqrt{x}$, $y = 3$ and $x = 0$ is revolved about the y-axis.

$y = \sqrt{x}$

Solve for x (get in terms of y)

$x = y^2$



$$\int_0^3 \pi (y^2)^2 \, dy = \pi \int_0^3 y^4 \, dy$$

$$= \frac{\pi}{5} y^5 \Big|_0^3 = \frac{243\pi}{5}$$