Anti-Derivative (U-Sub) WS

1) $\int (4x-3)^9\,dx = \int u^9 \left(\frac{du}{4}\right) = \frac{1}{4}\int u^9\,du$

$u = 4x-3$
$du = 4\,dx$
$dx = \frac{du}{4}$

$= \frac{1}{4}\left(\frac{u^{10}}{10}\right) + C$

$= \boxed{\frac{(4x-3)^{10}}{40} + C}$

Note: This problem can be done <u>without</u> u-sub.

2) $\int e^{2x}\,dx = \int e^u \left(\frac{du}{2}\right) = \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C$

$= \boxed{\frac{1}{2}e^{2x} + C}$

$u = 2x$
$du = 2\,dx$
$dx = \frac{du}{2}$

Note: can <u>also</u> be done w/o u-sub.

3) $\int \frac{1}{2x}\,dx = \frac{1}{2}\int \frac{1}{x}\,dx = \frac{1}{2}\ln|x| + C$

4) $\int t\sqrt{7t^2+12}\,dt = \int \cancel{t}\sqrt{u}\left(\frac{du}{14\cancel{t}}\right) = \frac{1}{14}\int u^{1/2}\,du$

$u = 7t^2+12$
$du = 14t\,dt$
$dt = \frac{du}{14t}$

$= \frac{1}{14}\left(\frac{u^{3/2}}{\frac{3}{2}}\right) + C$

$= \frac{1}{\cancel{14}_7}\left(\frac{\cancel{2}^1 u^{3/2}}{3}\right) + C$

$= \boxed{\frac{(7t^2+12)^{3/2}}{21} + C}$

5) $\int \frac{y}{\sqrt{4-5y^2}}\,dy = \int \frac{\cancel{y}}{\sqrt{u}}\left(\frac{du}{-10\cancel{y}}\right) = -\frac{1}{10}\int u^{-1/2}\,du$

$u = 4-5y^2$
$du = -10y\,dy$
$dy = \frac{du}{\ldots}$

$= -\frac{1}{10}\left(\frac{u^{1/2}}{\frac{1}{2}}\right) + C$

$= \boxed{-\frac{1}{\ldots}(4-5y^2)^{1/2} + C}$

6) $\int x^3 e^{x^4}\,dx = \int \cancel{x^3} e^u \left(\frac{du}{4\cancel{x^3}}\right) = \frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + C$

$\boxed{= \frac{1}{4}e^{x^4} + C}$

$u = x^4$
$du = 4x^3\,dx$
$dx = \frac{du}{4x^3}$

7) $\int \frac{t}{t^4+1}\,dt = \int \frac{t}{(t^2)^2+1}\,dt = \int \frac{\cancel{t}}{u^2+1}\left(\frac{du}{2\cancel{t}}\right) = \frac{1}{2}\int \frac{1}{u^2+1}du$

$u = t^2$
$du = 2t\,dt$
$dt = \frac{du}{2t}$

$= \frac{1}{2}\tan^{-1}u + C$

$\boxed{= \frac{1}{2}\tan^{-1}(t^2) + C}$

8) $\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx = \int \frac{\cancel{e^x+e^{-x}}}{u}\left(\frac{du}{\cancel{e^x+e^{-x}}}\right) = \int \frac{1}{u}\,du$

$u = e^x - e^{-x}$
$du = e^x + e^{-x}\,dx$
$dx = \frac{du}{e^x+e^{-x}}$

$= \ln|u| + C$

$\boxed{= \ln\left|e^x - e^{-x}\right| + C}$

9) $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x}}\,dx = \int \frac{\sec^2(u)}{\cancel{\sqrt{x}}}\left(2\cancel{\sqrt{x}}\,du\right) = 2\int \sec^2 u\,du$

$u = \sqrt{x} = x^{1/2}$
$du = \frac{1}{2}x^{-1/2}\,dx = \frac{1}{2\sqrt{x}}\,dx$
$dx = 2\sqrt{x}\,du$

$= 2\tan u + C$

$\boxed{= 2\tan(\sqrt{x}) + C}$

0) $\int \cos 2t \sin^5 2t\,dt = \int \cancel{\cos 2t}\,(u)^5\left(\frac{du}{2\cancel{\cos 2t}}\right) = \frac{1}{2}\int u^5\,du$

$u = \sin 2t$
$du = \cos 2t\,(2)\,dt$
$dt = \frac{du}{2\cos 2t}$

$= \frac{1}{2}\left(\frac{u^6}{6}\right) + C$

$= \frac{1}{}\sin^6 2t + C$

11) $\int \tan^3 5\theta \sec^2 5\theta \, d\theta = \int u^3 \cancel{\sec^2 5\theta}\left(\frac{du}{5\cancel{\sec^2 5\theta}}\right)$

$u = \tan 5\theta$

$du = \sec^2 5\theta \cdot 5\, d\theta$

$d\theta = \dfrac{du}{5\sec^2 5\theta}$

$= \frac{1}{5}\int u^3\, du$

$= \frac{1}{5}\left(\frac{u^4}{4}\right) + C$

$= \boxed{\dfrac{\tan^4 5\theta}{20} + C}$

12) $\int \frac{\sin x}{\cos^2 x + 1}\,dx = \int \frac{\cancel{\sin x}}{u^2+1}\left(\frac{du}{-\cancel{\sin x}}\right) = -\int \frac{1}{u^2+1}\,du$

$u = \cos x \, dx$

$du = -\sin x \, dx$

$dx = \dfrac{du}{-\sin x}$

$= -\tan^{-1} u + C$

$\boxed{= -\tan^{-1}(\cos x) + C}$

13) tangent $\longrightarrow$ 1st derivative

$y = 2x^3 - 3x^2 - 12x + 20$

$m_{tan} = \frac{dy}{dx} = 6x^2 - 6x - 12 = 0$ divide by 6

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

$\perp$ to y-axis

y axis: m = undef.

$\therefore$ a line $\perp$ to y-axis has a slope of 0.

So, make $\frac{dy}{dx} = 0$

To find points, plug back into original function.

(2,0) $y = 2(2)^3 - 3(2)^2 - 12(2) + 20 = 16 - 12 - 24 + 20 = 0$

$y = 2(-1)^3 - 3(-1)^2 - 12(-1) + 20 = -2 - 3 + 12 + 20 = 27$

$\boxed{(2,0) \text{ and } (-1,27)}$

14) $3x^2 - 4y^2 + y = 9$ tangent $\longrightarrow$ 1st derivative

$6x - 8y\frac{dy}{dx} + \frac{dy}{dx} = 0$

$$-8y\frac{dy}{dx}+\frac{dy}{dx}=-6x$$

$$\frac{dy}{dx}(-8y+1)=-6x$$

$$\frac{dy}{dx}=\frac{-6x}{-8y+1}$$

$$m_{tan} @ (2,1)=\left.\frac{dy}{dx}\right|_{(x,y)=(2,1)}=\frac{-6(2)}{-8(1)+1}=\frac{-12}{-7}=\boxed{\frac{12}{7}}$$

15) $y=\left(\frac{2x+8}{x^2-10x}\right)^5$

$$\frac{dy}{dx}=5\left(\frac{2x+8}{x^2-10x}\right)^4\left[\frac{2(x^2-10x)-(2x-10)(2x+8)}{(x^2-10x)^2}\right]$$

<u>don't</u> simplify.

16) $s(t)=t^4-4t^2+4,\quad t\geq 0$

A) $v(t)=s'(t)=4t^3-8t$

$a(t)=v'(t)=s''(t)=12t^2-8$

B) position @ $t=1$: $s(1)=1^4-4(1)^2+4=1$ ft

velocity @ $t=1$: $v(1)=4(1)^3-8(1)=-4$ ft/sec

speed $=|\text{velocity}|=|-4|=4$ ft/sec

absolute value

acceleration @ $t=1$: $a(1)=12(1)^2-8=4$ ft/sec^2

C) particle stops when $V(t)=0$

$$4t^3-8t=0$$
$$4t(t^2-2)=0$$

$4t=0$ $\quad t^2-2=0$

$t=0$ $\quad t=\pm\sqrt{2}$ $\quad$ but $t\geq 0$ so $t\neq-\sqrt{2}$

17.

$$f(x) = \begin{cases} x^2 - 5, & x \le 3 \\ x + 2, & x > 3 \end{cases}$$

A) $\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} x^2 - 5 = 3^2 - 5 = 4$

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} x + 2 = 3 + 2 = 5$

$\therefore$ jump disc. @ $x = 3$

$\therefore \lim_{x \to 3} f(x)$ DNE

B)

$$f'(x) = \begin{cases} 2x, & x \le 3 \\ 1, & x > 3 \end{cases}$$

c) $f'(3)$ DNE b/c a derivative does not exist at a discontinuity.

16) E) Distance traveled from $t = 0$ s to $t = 5$ s

Particle changes direction @ $t = \sqrt{2}$ s b/c $v(t) = (-)$ before $t = \sqrt{2}$ s and $v(t) = (+)$ after $t = \sqrt{2}$ s

$\therefore$ Distance traveled from 0 to 5 = $\underbrace{\text{Distance traveled from 0 to } \sqrt{2}}_{\text{Negative dist.}}$ + $\underbrace{\text{Distance traveled from } \sqrt{2} \text{ to } 5}_{\text{Positive distance}}$

Neg. dist.: $s(t_f) - s(t_i) = s(\sqrt{2}) - s(0) = 0 - 4 = -4 \rightarrow$ traveled 4 feet

Pos. dist.: $s(t_f) - s(t_i) = s(5) - s(\sqrt{2}) = 529 - 0 = 529$ ft

$\boxed{533}$ (feet)