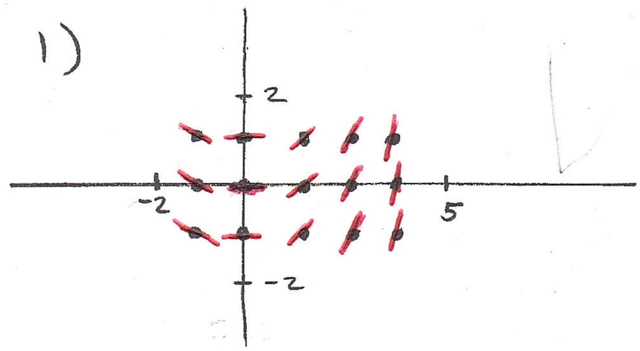
Slope fields WS 1

Directions: Solve each differential equation for an equation of y in terms of x using the given initial condition. Then draw a slope field for each differential equation using the given coordinate plane.

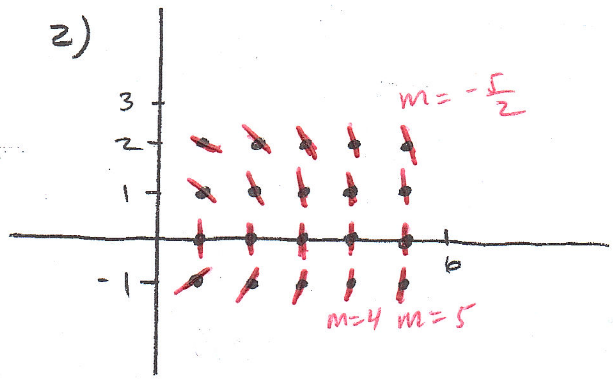
1) $\frac{dy}{dx} = \frac{x}{2}$, $y(1) = 6$

1)



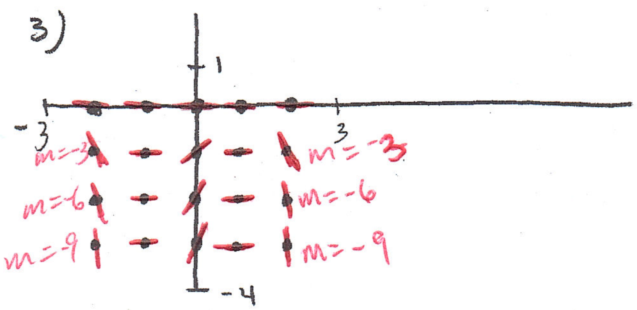
2) $\frac{dy}{dx} = -\frac{x}{y}$, $y(2) = -4$

2)



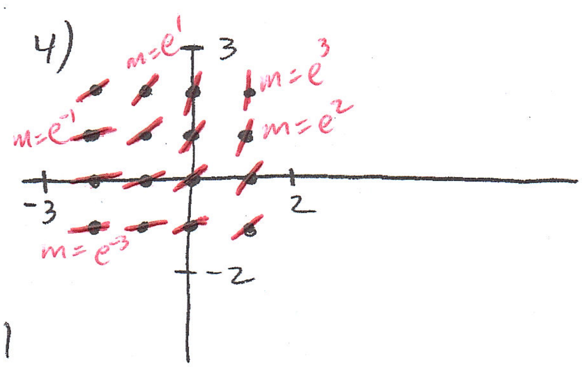
3) $\frac{dy}{dx} = x^2y - y$, $y(3) = 1$

3)



4) $\frac{dy}{dx} = e^{x+y}$, $y(1) = -1$

4)



5) $\frac{dy}{dx} = \frac{xy}{\ln(y^2)}$, $y(2) = 1$

make your own graph for #5 with 20 dots.

Slope fields WS1

1) $\frac{dy}{dx} = \frac{x}{2}$, $y(1) = 6$

$\int dy = \int \frac{x}{2}\,dx$

$y = \frac{x^2}{4} + C$

$6 = \frac{1}{4} + C$

$C = 5\frac{3}{4}$

$\boxed{\therefore y = \frac{x^2}{4} + \frac{23}{4}}$

graph shown below

2) $\frac{dy}{dx} = -\frac{x}{y}$; $y(2) = -4$

$\int y\,dy = \int -x\,dx$

$\frac{y^2}{2} = -\frac{x^2}{2} + C$

$\frac{(-4)^2}{2} = -\frac{(2)^2}{2} + C$

$\frac{16}{2} = -\frac{4}{2} + C$

$8 = -2 + C$

$C = 10$

$\therefore \frac{y^2}{2} = -\frac{x^2}{2} + 10$

or $\boxed{y = \pm\sqrt{20 - x^2}}$

3) $\frac{dy}{dx} = x^2 y - y$, $y(3) = 1$

$\frac{dy}{dx} = y(x^2 - 1)$

$\int \frac{dy}{y} = \int x^2 - 1\,dx$

$\ln y = \frac{x^3}{3} - x + C$

$\ln 1 = \frac{3^3}{3} - 3 + C$

$0 = 9 - 3 + C$

$C = -6$

$\therefore \ln y = \frac{x^3}{3} - x - 6$

or

$\boxed{y = e^{\frac{x^3}{3} - x - 6}}$

4) $\frac{dy}{dx} = e^{x+y}$, $y(1) = -1$

$\frac{dy}{dx} = e^x \cdot e^y$

$\frac{1}{e^y} dy = e^x dx$

$\int e^{-y} dy = \int e^x dx$

$-e^{-y} = e^x + c$

$-e^1 = e^1 + c$

$-2e = c$

$\therefore -e^{-y} = e^x - 2e$

or

$e^{-y} = 2e - e^x$

$\ln(e^{-y}) = \ln(2e - e^x)$

$-y = \ln(2e - e^x)$

$\boxed{y = -\ln(2e - e^x)}$

5) $\frac{dy}{dx} = \frac{xy}{\ln(y^2)}$ $\quad y(2) = 1$

$\int \frac{\ln(y^2)}{y} dy = \int x\, dx$

$u = \ln(y^2)$

$du = \frac{1}{y^2}(2y)\, dy = \frac{2}{y} dy$

$dy = \frac{y}{2} du$

$\int \frac{u}{y}\left(\frac{y}{2}\right) du = \frac{x^2}{2} + c$

$\frac{1}{2} \int u\, du = \frac{x^2}{2} + c$

$\frac{1}{2}\left(\frac{u^2}{2}\right) = \frac{x^2}{2} + c$

$\frac{1}{4}(\ln(y^2))^2 = \frac{x^2}{2} + c$

$\frac{1}{4}(\ln 1)^2 = \frac{2^2}{2} + c$

$0 = 2 + c$

$c = -2$

$\frac{1}{4}(\ln(y^2))^2 = \frac{x^2}{2} - 2$

or

$(\ln(y^2))^2 = 2x^2 - 8$

$\ln(y^2) = \pm\sqrt{2x^2 - 8}$

$y^2 = e^{\pm\sqrt{2x^2-8}}$

$\boxed{y = \pm\sqrt{e^{\pm\sqrt{2x^2-8}}}}$