2010B
#2

Review WS 9

a) $g'(x) = \sin\left(x + \frac{1}{x}\right) = 0$

$x = 0.163$ and $x = 0.359$

b) $g''(x) = \left(1 - \frac{1}{x^2}\right)\cos\left(x + \frac{1}{x}\right) = 0$

$x = .129, \; x = .223$

concave down on $(.129, .223)$ because $g''(x)$ is neg.

c) point

$(.3, g(.3))$

slope

$g'(.3) = -.472$

$$\int_{.3}^{1} g'(x)\,dx = g(1) - g(.3)$$

$.454 = 2 - g(.3)$

$g(.3) = 2 - .454 = 1.546$

$(.3, 1.546)$

$y - y_1 = m(x - x_1)$

$y - 1.546 = -0.472(x - 0.3)$

$y = -0.472(x - 0.3) + 1.546$

d) below because $g(x)$ is concave up for $0.3 < x < 1$.

2010B #3

a) $\int_0^{12} P(t)\,dt = A_1 + A_2 + A_3$

where $A_1 = l \cdot w = (46)(4) = 184$

$A_2 = l \cdot w = (57)(4) = 228$

$A_3 = l \cdot w = (62)(4) = 248$

$\boxed{660 \text{ ft}^3}$

b) Units of answer: ft^3

$R(t) =$ ft^3/hour

total amt of water $= \int_0^{12} |R(t)|\,dt = \boxed{225.594 \text{ ft}^3}$

c) $V(t) = 1000 + \int_0^{12} P(t)\,dt - \int_0^{12} |R(t)|\,dt$

$= 1000 + 660 - 225.594 = \boxed{1434.406 \text{ ft}^3}$

$\boxed{1434 \text{ ft}^3}$

d) increasing $\longrightarrow$ 1st derivative

$V'(t) = P(t) - R(t)$

$V'(8) = P(8) - R(8) = 60 - 16.758 = \boxed{43.242 \text{ ft}^3/\text{hour}}$

Water level in the pool rising (i.e. increasing) $\longrightarrow \frac{dh}{dt} = ?$

$V = \pi r^2 h$ only h and V change with time.

$\frac{dV}{dt} = \pi r^2 \frac{dh}{dt}$

$43.242 = \pi (12)^2 \frac{dh}{dt} \Longrightarrow \frac{dh}{dt} = \frac{43.242}{144\pi} = \boxed{.096 \text{ ft/hour}}$

2008 #4

$X(0) = -2$

a) possible answers

$X(0) = -2$

$X(3) = ?$

$X(6) = ?$

$$\int v(t)\,dt = X(t)$$

$$\int_0^3 v(t)\,dt = X(3) - X(0) \quad \text{and} \quad \int_0^6 v(t)\,dt = X(6) - X(0)$$

$$-8 = X(3) + 2 \qquad\qquad -7 = X(6) + 2$$

$$X(3) = -10 \qquad\qquad X(6) = -9$$

The particle is farthest to the left @ $t = 3$ b/c from $0 \le t \le 6$ the particle's position @ $t = 3$ is -10.

b) $X(5) = ?$

$$\int_0^5 v(t)\,dt = X(5) - X(0)$$

$$-5 = X(5) + 2$$

$$X(5) = -7$$

The particle starts at $X(0) = -2$ ad then travels to the left until it reaches $X(3) = -10$. Then the particle travels to the right until it reaches $X(5) = -7$ ad then travels left until $X(6) = -9$

Since velocity is continuous, the particle is @ $x=-8$ 3 times.

c) On the interval $2<t<3$ the speed of the particle is decreasing b/c $v(t)$ is negative and $a(t)$ is positive.

d) The acceleration is negative from $t=0$ to $t=1$ and from $t=4$ to $t=6$ because the slopes of the tangent lines are negative on those intervals.