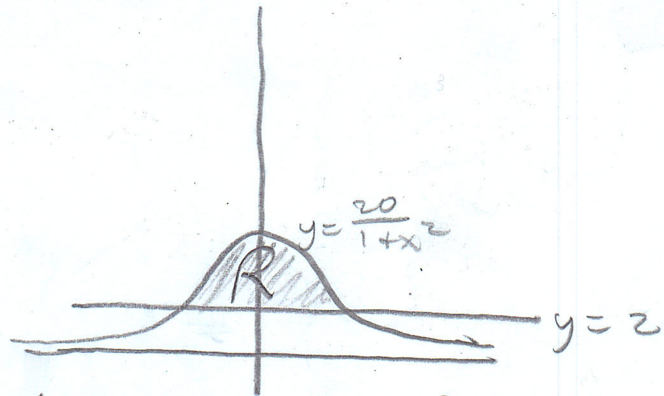
Review WS 8

2007 #1

$$y = \frac{20}{1+x^2}, \quad y = 2$$



a) Area = $\int$ top-bottom dx

Limits of integration: pts. of intersection (set function = to each other)

$$\frac{20}{1+x^2} = 2$$

$$20 = 2(1+x^2)$$
$$20 = 2 + 2x^2$$
$$18 = 2x^2$$
$$9 = x^2$$
$$x = \pm 3$$

$$\int_{-3}^{3} \frac{20}{1+x^2} - 2\, dx = 37.962$$

b) Revolve around x-axis means cross section is a washer.

$$V = \pi \int_{-3}^{3} R^2 - r^2\, dx = \pi \int_{-3}^{3} \left(\frac{20}{1+x^2}\right)^2 - (2)^2 dx = 1871.190$$

c) cross sections are semi-circles $\left(A = \frac{\pi r^2}{2}\right)$

$$\therefore \quad V = \int_{-3}^{3} \frac{\pi r^2}{2} dx = \frac{\pi}{2} \int_{-3}^{3} r^2 dx$$

The radius = $\frac{1}{2}$ of distance between curves = $\frac{1}{2}$(top-bottom)

$r = \frac{1}{2}\left(\frac{20}{1+x^2} - 2\right)$

$$\therefore \quad V = \frac{\pi}{2}\int_{-3}^{3}\left(\frac{1}{2}\left(\frac{20}{1+x^2}-2\right)\right)^2 dx = 174.268$$

2007 #2

a) $\int_0^7 f(t)\,dt = \int_0^7 100t^2 \sin\sqrt{t}\,dt = 8263.807$

$= 8264$ gallons

b) I like to compose one "big" function that represents the amount of water (in gallons) in the storage tank at any time, t.

$$W(t) = \underbrace{5000}_{\text{initial amount}} + \underbrace{\int_0^t f(t)\,dt}_{\text{amount entering tank}} - \underbrace{\int_0^t g(t)\,dt}_{\text{amount leaving tank}}$$

Decreasing $\longrightarrow$ take the derivative, critical pts, sign chart
① ② ③

① $W'(t) = f(t) - g(t)$

② $W'(t) = f(t) - g(t) = 0 \Longrightarrow f(t) = g(t)$

where does $f(t) = g(t)$
in other words, find the t values where the functions intersect.

$\Longrightarrow f(t) = g(t)$ @ $t = 1.617$
and $t = 5.076$

$t = 1.617$ and $t = 5.076$ are critical pts but not the only ones. Critical pts also occur where $W'(t)$ DNE.

And, $W'(t)$ DNE @ $t = 3$ b/c $W'(3) = f(3) - g(3)$ but $g(3)$ DNE.

All critical pts @ $t = 1.617, 3, 5.076$

(3)

1.617   3   5.076

0	2	4	6
−	+	−	+

I'll show you how to plug 0 and 4 in. You try the rest.

$W'(0) = f(0) - g(0) = 0 - 250$
$= -250$

$W'(4) = f(4) - g(4)$
$f(4) < g(4) \therefore W'(4) = -$

$\therefore$ The amount of water in the tank is decreasing from $0 < t < 1.617$ and $3 < t < 5.076$.

c) Amount of water in tank greatest $\rightarrow$ Absoute max.

Abs. max $\rightarrow$ (1) take der., (2) find critical pts, (3) sign chart, (4) find relative max, (5) test rel. max and endpts in orignal function

Steps 1, 2 and 3 are already done

Step (4) relative max @ $t = 3$ b/c $W'(t)$ changes from + to −

Find $W(0)$, $W(3)$, $W(7)$

$W(0) = 5000$

$$W(3) = 5000 + \int_0^3 100t^2 \sin(\sqrt{t}) - \int_0^3 250\,dt = 5126.591$$

$$W(7) = 5000 + \int_0^7 100t^2 \sin(\sqrt{t}) - \underbrace{\left(\int_0^3 250\,dt + \int_3^7 2000\,dt\right)}_{=\int_0^7 g(t)\,dt}$$

$$W(7) = 5000 + 8263.807 - 8750 = 4513.807 \text{ gallons}$$

The amount of water in the tank is greatest @ $t=3$, $W(3) = 5127$ gallons

2007 #3

a) $h(1) = f(g(1)) - 6 = f(2) - 6 = 9 - 6 = 3$

$h(3) = f(g(3)) - 6 = f(4) - 6 = -1 - 6 = -7$

since f and g are differentiable then they are continuous, thus, $h(x)$ is continuous.

$\therefore$ by Intermediate Value Th$^{\underline{m}}$ there exists an $1<r<3$ such that $h(r) = -5$.

b) since f and g are diff. then h is diff and continuous. Thus, the Mean Value Th$^{\underline{m}}$ says

n $1<c<3$, $h'(c) = \dfrac{h(3) - h(1)}{3-1} = \dfrac{-7-3}{2} = -5$

c) $W(x) = \int_1^{g(x)} f(t)\,dt$

$$W'(x) = \frac{d}{dx}\int_1^{g(x)} f(t)\,dt = f(g(x)) \cdot g'(x) - 0$$

$$W'(3) = f(g(3)) \cdot g'(3) = f(4) \cdot 2 = -1 \cdot 2 = -2$$

d) inverse(g^{-1}) | regular/original(g)

$x = 2$ | $y = 2$

$y = 1 \longleftarrow$ from table $x = 1$

point: $(2, 1)$

$$m_{tan} = \frac{d}{dx} g^{-1}(x) = \frac{1}{\frac{d}{dx} g(y)} = \frac{1}{g'(1)} = \frac{1}{5}$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{1}{5}(x - 2)$$

$$y = \frac{1}{5}(x-2) + 1$$