2010 #3

Review WS 6

a) $\int_0^3 r(t)\,dt$ = Area under curve of $r(t)$ from $t=0$ to $t=3$.

= Area of rectangle + Area of Δ + Area of trap.

$= 3(800) + \frac{1}{2}(1)(400) + \left(\frac{400+200}{2}\right)(2)$

$= 2400 + 200 + 600 = \boxed{3200 \text{ people arrive}}$

b) A function that represents the number of people in line at time t

$$\underbrace{700}_{\text{initial}} + \underbrace{\int_0^t r(t)\,dt}_{\text{people joining the line}} - \underbrace{800t}_{\text{people "exiting" the line}} = L(t)$$

$L'(t) = r(t) - 800 = 0$

$r(t) = 800$ @ $t = 3$ (from graph)

$r(2) - 800 = +400$ ← 2 | $\boxed{3}$ | 5 → $r(5) - 800 = -400$

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$\therefore$ from $t=2$ to $t=3$ the number of people in line is increasing.

c) Longest line → max. people in line

$L(t)$ is increasing from $t=0$ to $t=3$ and decreasing from $t=3$ to $t=8$ $\therefore$ The line is the longest @ $t=3$. $L(3) = 700 + \int_0^3 r(t)\,dt - 800(3) = 700 + 3200 - 2400$

$= \boxed{1500 \text{ people}}$

d) $700 + \int_0^t r(t)\,dt - 800t = 0$

2011B #5

a) $a(t) = v'(t)$

$$a(5) \approx \frac{V(10) - V(0)}{10 - 0} = \frac{2.3 - 2.0}{10} = \frac{.3}{10} = \boxed{.03\ m/s^2}$$

b) $\int_0^{60} |v(t)|\,dt$ = Total distance Ben rode in the 1st minute in meters.

$$\int_0^{60} |v(t)|\,dt = A_1 + A_2 + A_3$$
$$= (10)(2.0) + (30)(2.3) + (20)(2.5)$$
$$= 20 + 69 + 50 = \boxed{139 \text{ meters}}$$

c) $B(t)$ is continuous on $[40, 60]$ and diff. on $(40, 60)$.

$$\frac{B(60) - B(40)}{60 - 40} = \frac{49 - 9}{20} = 2 \quad \therefore \text{ By the Mean Value Th}^{m}.$$

There must be a value between 40 and 60 such that $v(t) = B'(t) = 2$.

d) $(L(t))^2 = 12^2 + (B(t))^2$

$$2 \cdot L(t) \cdot L'(t) = 2 \cdot B(t) \cdot B'(t)$$
$$2 \cdot L(40) \cdot L'(40) = 2 \cdot B(40) \cdot B'(40)$$
$$L'(40) = \frac{\cancel{2} \cdot B(40) \cdot B'(40)}{\cancel{2} \cdot L(40)} = \frac{B(40)\,v(40)}{L(40)} = \frac{9\,(2.5)}{\sqrt{144 + (9)^2}} = \frac{22.5}{\sqrt{225}} = \boxed{\frac{22.5}{15}\ m/sec}$$