2004

Review WS 5

3) $v(t) = 1 - \tan^{-1}(e^t)$

a) $a(2) = v'(2) = -0.133$

b) $v(2) = -0.436$

$\therefore$ speed is inc. b/c acceleration and velocity are both negative.

c) Particle reaches highest point when $v(t) = 0$

$$1 - \tan^{-1}(e^t) = 0$$
$$\tan^{-1}(e^t) = 1$$
$$e^t = \tan 1$$
$$t = \ln(\tan 1) = 0.443$$

$v(t)$ is positive for $0 < t < 0.443$ and

$v(t)$ is negative for $t > 0.443$

$\therefore$ the particle reaches its highest point @ $t = 0.443$

d) $s(2) = ?$, $s(0) = -1$

$$\int_0^2 v(t)\,dt = s(2) - s(0)$$

$$\int_0^2 v(t)\,dt = s(2) - (-1)$$

$$s(2) = \int_0^2 v(t)\,dt - 1 = -1.361$$

$v(2) = -0.436$ and $s(2) = -1.361$

moving down

already below origin

$\therefore$ particle is moving away @ $t = 2$.

2010 B

4) a) squirrel changes direction @ $t=9$ and $t=15$ b/c the velocity changes sign.

b) squirrel travels away from building A from $t=0$ to $t=9$ ~~and~~ from $t=15$ to $t=18$ b/c $v(t)$ is positive.

Squirrel travels toward building A from $t=9$ to $t=15$ b/c $v(t)$ is negative

$s(0)=0$

$s(9)=$ area under curve from $t=0$ to $t=9$

$= \left(\frac{5+9}{2}\right)(20)=140$

$s(15)=$ Area under curve from $t=0$ to $t=9$

"minus" $-$ Area under curve from $t=9$ to $t=15$

$s(15)=140-\left(\frac{6+4}{2}\right)(10)=90$

$s(18)=$ Area under curve from $t=0$ to $t=9$

"minus" $-$ Area und curve from $t=9$ to $t=15$

"plus" $+$ Area under curve from $t=15$ to $t=18$

$s(18)=140-50+\left(\frac{2+3}{2}\right)(10)=115$

$\therefore$ the squirrel is furthest from building A @ $t=9$ with a distance of 140.

c) Total distance = Area under curve from $t=0$ to $t=9$

"All positive" $\left\{ \begin{array}{l} + \text{ Area under curve from } t=9 \text{ to } t=15 \\ + \text{ Area under curve from } t=15 \text{ to } t=18 \end{array} \right.$

Total distance $= 140 + 50 + 25 = 215$

d) $a(t) = v'(t) =$ slopes of tangent lines of $v(t)$.

From $7 < t < 10$ $v'(t)$ is constant

$$v'(t) = \frac{20-(-10)}{7-10} = \frac{30}{-3} = -10$$

$\therefore$ from $7 < t < 10$ $a(t) = -10$

From $7 < t < 10$ $v(t)$ is a line with $m = -10$ point $(7, 20)$

$$y - y_1 = m(x - x_1)$$
$$y - 20 = -10(x-7)$$
$$y - 20 = -10x + 70$$
$$y = -10x + 90 \Longrightarrow v(t) = -10t + 90$$

Distance from building A = $s(t)$ for $7 < t < 10$

$$\int_7^t v(t)\,dt = s(t) - s(7)$$

$$s(t) = \int_7^t v(t)\,dt + s(7)$$

$s(7)$ = Area under curve from $t=0$ to $t=7$
$= \left(\frac{7+5}{2}\right)(20) = 120$

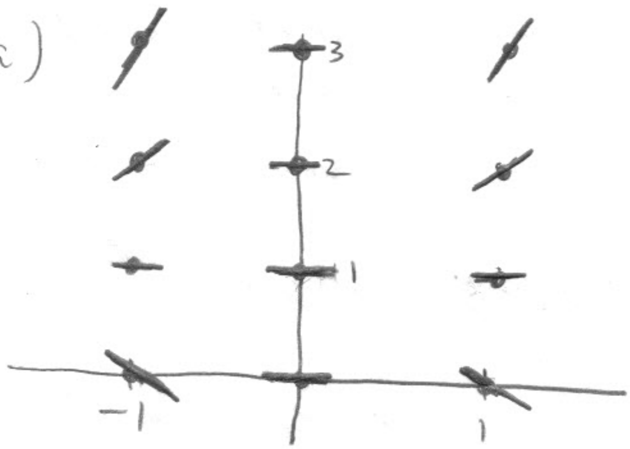
$$= \int_7^t -10t + 90\,dt + 120$$

$$= -5t^2 + 90t \Big|_7^t + 120$$

$$= -5t^2 + 90t - (-245 + 630) + 120 = -5t^2 + 90t - 265$$

2004

6) a)



b) $\frac{dy}{dx} = x^2(y-1) > 0$

when x^2 and $y-1$ are the same sign but x^2 is always positive, so need to find where $y-1>0$

$y>1$ and $x \neq 0$

$\therefore$ the slopes are positive for any point with $y>1$.

c) $\frac{dy}{dx} = x^2(y-1)$, $f(0)=3$

$\int \frac{1}{y-1}\,dy = \int x^2\,dx$

$\ln|y-1| = \frac{x^3}{3} + C$

$\ln|3-1| = \frac{0^3}{3} + C$

$C = \ln 2$

$\ln|y-1| = \frac{x^3}{3} + \ln 2$

$y-1 = e^{\frac{x^3}{3}+\ln 2} = e^{\frac{x^3}{3}} \cdot e^{\ln 2} = 2e^{\frac{x^3}{3}}$

$y = 2e^{\frac{x^3}{3}} + 1$

2010 B

6) $P(t) = 2\cos\left(\frac{\pi}{4}t\right)$ $\qquad R(t) = t^3 - 6t^2 + 9t + 3$

a) $R'(t) = 3t^2 - 12t + 9 = 0$

$t^2 - 4t + 3 = 0$

$(t-3)(t-1) = 0$

$t = 1, 3$

0	$\boxed{1}$	2	$\boxed{3}$	5
+		−		+

$\therefore$ Particle R is moving to the right for $0 \le t \le 1$ and $3 \le t \le 6$ b/c $R'(t)$ is positive

b) $P'(t) = -2\sin\left(\frac{\pi}{4}t\right) \cdot \left(\frac{\pi}{4}\right) = -\frac{\pi}{2}\sin\left(\frac{\pi}{4}t\right) = 0$

$\Rightarrow \sin\left(\frac{\pi}{4}t\right) = 0$

$\frac{\pi}{4}t = 0$, $\frac{\pi}{4}t = \pi$, $\frac{\pi}{4}t = 2\pi$

$t = 0$ $\qquad t = 4$ $\qquad t = 8 \leftarrow$ too big

$\boxed{0}$	1	$\boxed{4}$	6
	−		+

	$\boxed{0}$		$\boxed{1}$		$\boxed{3}$		$\boxed{4}$		$\boxed{6}$
$R(t)$		+		−		+		+	
$P(t)$		—		—		—		+	

$\therefore$ Particle P and particle R are traveling in opposite directions for $0 < t < 1$ and $3 < t < 4$.

c) Particle P: $p(t) = 2\cos\left(\frac{\pi}{4}t\right)$

$$v(t) = -\frac{\pi}{2}\sin\left(\frac{\pi}{4}t\right)$$

$$a(t) = -\frac{\pi}{2}\cos\left(\frac{\pi}{4}t\right)\cdot\left(\frac{\pi}{4}\right)$$

$$a(t) = -\frac{\pi^2}{8}\cos\left(\frac{\pi}{4}t\right)$$

$$a(3) = -\frac{\pi^2}{8}\cos\left(\frac{3\pi}{4}\right) = -\frac{\pi^2}{8}\left(-\frac{\sqrt{2}}{2}\right) = \frac{\pi^2\sqrt{2}}{16}$$

$$v(3) = -\frac{\pi}{2}\sin\left(\frac{3\pi}{4}\right) = -\frac{\pi}{2}\left(\frac{\sqrt{2}}{2}\right) = -\frac{\pi\sqrt{2}}{4}$$

Since $a(3)$ is positive and $v(3)$ is neg then Particle P is slowing down

d) $$\frac{1}{3-1}\int_1^3 |P(t) - R(t)|\,dt = \frac{1}{2}\int_1^3 |P(t) - R(t)|\,dt$$