Review WS 4

2011 B #6

1) a) $\int_{-2\pi}^{4\pi} f(x)\,dx = \underbrace{\int_{-2\pi}^{4\pi} g(x)\,dx}_{\text{Area between } g(x) \text{ and x-axis}} - \int_{-2\pi}^{4\pi} \cos\left(\frac{x}{2}\right)dx$

$\Rightarrow = \frac{1}{2}bh - \left.\frac{\sin\frac{x}{2}}{\frac{1}{2}}\right|_{-2\pi}^{4\pi} = \frac{1}{2}(6\pi)(2\pi) - \left(2\sin\frac{4\pi}{2} - 2\sin\frac{-2\pi}{2}\right)$

$\Rightarrow = 6\pi^2 - (2(0) - 2(0)) = 6\pi^2$

b) f has a critical pt @ $x=0$ b/c $g'(0)$ is undef.

$f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right) = \begin{cases} 1 + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } -2\pi < x \le 0 \\ -\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } 0 \le x < 4\pi \end{cases}$

b/c $g'(x) = m_{tan} = \begin{cases} 1 & -2\pi < x \le 0 \\ -\frac{1}{2} & 0 \le x < 4\pi \end{cases}$

critical pt. $(f' = 0)$

top: $1 + \frac{1}{2}\sin\left(\frac{x}{2}\right) = 0$

$\sin\left(\frac{x}{2}\right) = -2$

No solution b/c $-1 \le \sin\left(\frac{x}{2}\right) \le 1$

bottom: $-\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right) = 0$

$\sin\left(\frac{x}{2}\right) = 1$

$\frac{x}{2} = \frac{\pi}{2}$ $\quad \frac{x}{2} = -\frac{3\pi}{2}$

$x = \pi$ $\quad x = \cancel{-3\pi}$ too small

$\therefore$ only critical pts for f are $x = 0$ b/c $f'(0)$ is undef. and $x = \pi$ b/c $f'(\pi) = 0$.

c) $h(x) = \int_0^{3x} g(t)\,dt$

$h'(x) = \frac{d}{dx}\int_0^{3x} g(t)\,dt = g(3x)\cdot 3 = 3g(3x)$

$h'(-\frac{\pi}{3}) = 3g(3(-\frac{\pi}{3})) = 3g(-\pi) = 3(\pi) = \boxed{3\pi}$

from graph

2011 #2

2) a) $H'(3.5) = \frac{H(5) - H(2)}{5-2} = \frac{52-60}{3} = \boxed{-\frac{8}{3}\ {}^\circ C/min}$

b) $\frac{1}{10}\int_0^{10} H(t)\,dt$ is the average temperature of the tea in °C/min from $t=0$ to $t=10$ min.

$\int_0^{10} H(t)\,dt = A_1 + A_2 + A_3 + A_4$

$A_1 = \left(\frac{b_1+b_2}{2}\right)h = \left(\frac{66+60}{2}\right)(2) = 126$

$A_2 = \left(\frac{60+52}{2}\right)(3) = 168$

$A_3 = \left(\frac{52+44}{2}\right)(4) = 192$

$A_4 = \left(\frac{44+43}{2}\right)(1) = 43.5$

$\therefore \frac{1}{10}\int_0^{10} H(t)\,dt = \frac{1}{10}(529.5) = 52.95$

c) $\int_0^{10} H'(t)\,dt = H(10) - H(0) = 43 - 66 = -23\ {}^\circ C$

The net change in temperature in the first ten minutes in °C.

(2)

d) $B'(t) = -13.84e^{-0.173t}$

$\int B'(t)\,dt = B(t)$

$\int_0^{10} B'(t)\,dt = B(10) - B(0)$

$\int_0^{10} -13.84e^{-0.173t}\,dt = B(10) - 100$

$-65.817 = B(10) - 100$

$B(10) = 34.183°C$

The temp. of the tea @ $t=10$ is $H(10) = 43$.

The biscuits are $43°C - 34.183°C = 8.817°C$ cooler than the tea.

2011 #5

3) a) Point

$W(0) = 1400$

$(0, 1400)$

Slope

$\frac{dw}{dt} = \frac{1}{25}(W - 300)$

$\frac{dw}{dt} = \frac{1}{25}(1400 - 300) = 44$

$\underset{\downarrow}{W}$ $\underset{\downarrow}{t}$

$y - y_1 = m(x - x_1)$

$W - 1400 = 44(t - 0)$

$W = 44t + 1400$

$W(t) = 44t + 1400$

$W(\frac{1}{4}) = 44(\frac{1}{4}) + 1400 = \boxed{1411 \text{ tons}}$

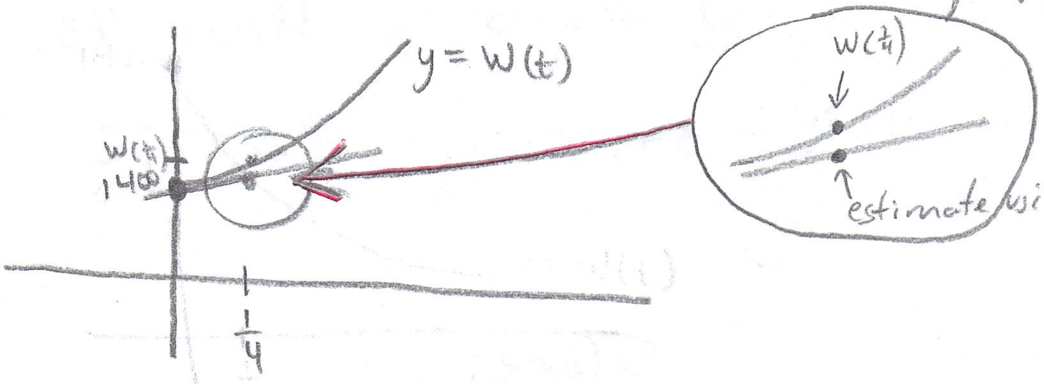
b) $\dfrac{dW}{dt} = \dfrac{1}{25}(W-300) = \dfrac{1}{25}W - 12$

$$\frac{d^2W}{dt^2} = \frac{1}{25}\frac{dW}{dt} = \frac{1}{25}\left(\frac{1}{25}(W-300)\right) = \frac{1}{625}(W-300)$$

$$\left.\frac{d^2W}{dt^2}\right|_{t=\frac{1}{4}} = \frac{1}{625}(1411-300) = + \quad , \therefore W \text{ is concave up @ } t = \tfrac{1}{4}$$

Since W is concave up @ $t = \frac{1}{4}$ then 1411 tons is an underestimate

Let me illustrate this with a graph



Tangent lines to curves that are concave up are always an underest.

c) $\dfrac{dW}{dt} = \dfrac{1}{25}(W-300)$

$$\frac{dW}{W-300} = \frac{1}{25}dt$$

$$\int \frac{dW}{W-300} = \frac{1}{25}\int dt$$

$$\ln|W-300| = \frac{1}{25}t + c$$

$$\ln(1400-300) = \frac{1}{25}(0) + c$$

$$c = \ln 1100$$

don't need absolute value anymore b/c $W(0) = +$ and W is inc. function

$$\therefore \ln(W-300) = \frac{1}{25}t + \ln 1100$$

$$e^{\ln(W-300)} = e^{\frac{1}{25}t + \ln 1100}$$

$$W - 300 = e^{\frac{1}{25}t} \cdot e^{\ln 1100}$$

$$W = 1100e^{\frac{1}{25}t} + 300$$