Review WS 2

1) $\frac{dV}{dt} = 10 \text{ in}^3/\text{min}$

a) Coffee pot is a cylinder.

$V_{cylinder} = \pi r^2 h$

the height changes with time but the radius does not.

$\therefore \frac{dV}{dt} = \underbrace{\pi r^2}_{\text{coefficients}} \frac{dh}{dt}$

$10 = \pi (3)^2 \frac{dh}{dt}$

$$\boxed{\frac{dh}{dt} = \frac{10}{9\pi} \text{ in/min}}$$

b) $V_{cone} = \frac{1}{3}\pi r^2 h$

r and h change with time.

$\frac{dV}{dt} = \frac{1}{3}\pi \left(2r\frac{dr}{dt}\cdot h + \frac{dh}{dt}\cdot r^2\right)$ ← too many variables

$h = 6,\ r = 3,\ \frac{dV}{dt} = 10,\ \frac{dh}{dt} = ?$, but $\frac{dr}{dt} = ?$

so, $\frac{r}{h} = \frac{3}{6}$

$6r = 3h$

$r = \frac{h}{2}$ → $V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 h$

$V = \frac{1}{3}\pi\left(\frac{h^2}{4}\right)h$

$V = \frac{\pi}{12}h^3$

$$\frac{dv}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$$

$$-10 = \frac{\pi}{4}(5)^2\frac{dh}{dt}$$

$$-10 = \frac{25\pi}{4}\frac{dh}{dt}$$

$$\frac{dh}{dt} = -\frac{40}{25\pi} = \boxed{-\frac{8}{5\pi} \text{ in/min}}$$

2) Graph is f' (1st derivative) 2004 Form B #4

a) Points of inflection occur where the 2nd derivative changes signs.

f'' changes from + to − @ $x=1$, from − to + @ $x=3$. ∴ the x-coordinates of the inflection pts are $x=1$ and $x=3$.

b) Minimums occur where the 1st derivative changes from − to +. Maximums occur where the 1st derivative changes from + to −.

There is an absolute min @ $x=4$ b/c f' is − from $x=-1$ to $x=4$ which means that f is decreasing on $[-1, 4]$.

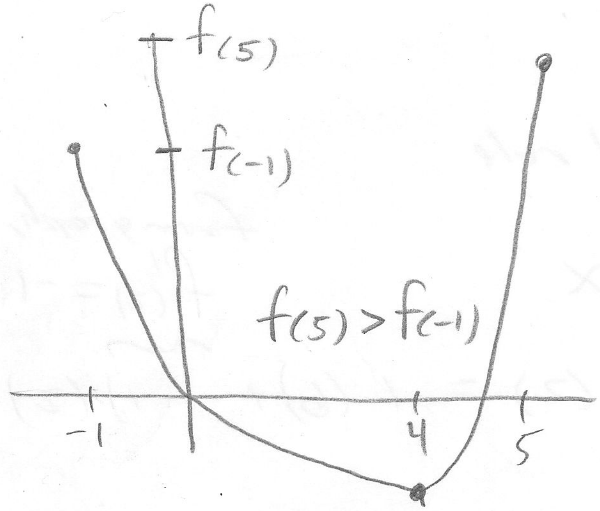
Finding the absolute maximum is trickier. Looking at the graph, f' is + from $x=4$ to $x=5$

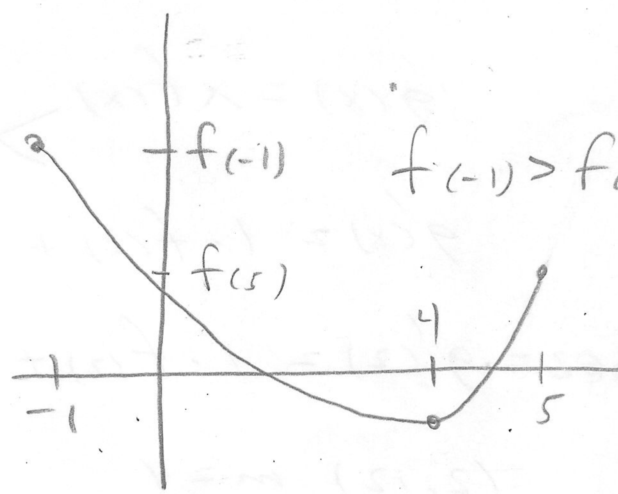
Thus, f is increasing on $[4, 5]$.

$\therefore$ There may be an absolute maximum @ $x = 5$. However an absolute max & min can also occur at the endpoints of the graph ie $x = -1$.

2 possible cases for f.

Case 1: or Case 2:





We can't find $f(5)$ or $f(-1)$, but

since $f = \int f'$ part 2 of FTC

then $f(5) - f(-1) = \underbrace{\int_{-1}^{5} f'(t)\,dt}$ part 1 of FTC

represents the area between $f'(t)$ and the x-axis for $x = -1$ to $x = 5$

Since most of the area is under the x-axis then $f(5) - f(-1) < 0$ which means that
$+f(-1)$ $+f(-1)$
$f(-1) > f(5)$. $\therefore$ Absolute maximum @ $x = -1$.

c) $g(x) = x\,f(x)$

equation of a line needs a point and a slope.

point: $x=2$ b/c $f(2)=6$

$y = g(2) = 2f(2) = 2(6) = 12$

$(2, 12)$

slope: 1st derivative @ $x=2$

$g(x) = x\,f(x)$ > product rule

$g'(x) = 1\cdot f(x) + f'(x)\cdot x$

from graph, $f'(2) = -1$

slope @ 2 $= g'(2) = 1\cdot f(2) + f'(2)\cdot(2) = 1\cdot(6) + (-1)(2) = 4$

$(2, 12)$ $m = 4$

$y - y_1 = m(x - x_1)$

$y - 12 = 4(x - 2)$

$y = 4x + 4$

3) $V(t) = -2 + (t^2+3t)^{6/5} - t^3 = Y_1$ ← calculator

$s(0) = 10$, when $t=0$ the particle's position is 10.

a) $2 \le t \le 4$

speed $= |v(t)| = 2 \Rightarrow v(t) = \pm 2$

$2 = -2 + (t^2+3t)^{6/5} - t^3$ (can't solve on paper)

to solve using a calculator, you have to make equation equal to zero, then graph and find zeros

$$\begin{aligned} 2 &= -2 + (t^2+3t)^{6/5} - t^3 \\ -2 & \quad -2 \\ 0 &= -4 + (t^2+3t)^{6/5} - t^3 \end{aligned}$$

$t = 3.128$

Now, $-2 = -2 + (t^2+3t)^{6/5} - t^3$

$+2 \quad +2$

$0 = (t^2+3t)^{6/5} - t^3$

$t = 3.473$

the particle's speed equals 2 when $t = 3.128$ and $t = 3.473$.

b) $\int v(t)\,dt = s(t) + c$

$\int_0^t v(x)\,dx = s(t) - s(0)$

$+s(0) \qquad\qquad +s(0)$

$s(0) + \int_0^t v(x)\,dx = s(t) \Rightarrow s(t) = 10 + \int_0^t v(x)\,dx$

So, $s(5) = 10 + \int_0^5 v(x)\,dx = -9.207$

calculator
(-19.207)

c) $v(t) = 0$

$-2 + (t^2 + 3t)^{6/5} - t^3 = 0$ (graph and find zeros)

$t = 0.536$, $t = 3.318$

the particle changes direction @ $t = 0.536$ b/c the velocity changes from neg to pos and it also changes direction @ $t = 3.318$ b/c the velocity changes from + to −.

d) speed inc / dec is the same as speeding up/slowing down.

$v(4) = -11.476$

$a(4) = -22.296$

Since the velocity and acceleration are both negative then the particle's speed is inc @ $t = 4$