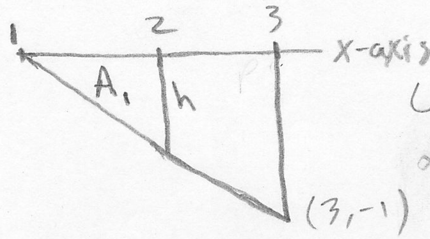
Review WS 11

3) $g(x) = \int_1^x f(t)\,dt$

a) $g(2) = \int_1^2 f(t)\,dt$ Area between curve and x-axis from $x=1$ to $x=2$.

picture from graph,



Use similar triangles to find height of ΔA_1.

$\frac{1}{h} = \frac{2}{1}$

$2h = 1$

$h = \frac{1}{2}$

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(1)(\frac{1}{2}) = \frac{1}{4}$

$\boxed{\therefore g(2) = -\frac{1}{4}}$

$g(-2) = \int_1^{-2} f(t)\,dt = -\int_{-2}^{1} f(t)\,dt$ Area between curve and x-axis from $x=-2$ to $x=1$

$-\left(\frac{1}{2}bh + -\left(\frac{\pi r^2}{2}\right)\right) = -\left(\frac{1}{2}(1)(3) - \frac{\pi(1)^2}{2}\right)$

negative b/c area is under the x-axis

Area of Δ

Area of semi ⊙

$= \boxed{-\left(\frac{3}{2} - \frac{\pi}{2}\right)}$

b) $g(x) = \int_1^x f(t)\,dt$

$g'(x) = \frac{d}{dx}\int_1^x f(t)\,dt = f(x)$

$g'(-3) = f(-3) = \boxed{2}$

y value for $x=-3$ on the graph of $f(x)$.

$g'(x) = f(x)$

$g''(x) = f'(x)$

$g''(-3) = f'(-3) =$ slope of tangent line to $y=f(x)$ @ $x=-3$

$$m_{tan} = \frac{3-1}{-2-(-4)} = \frac{2}{2} = 1$$

$$\boxed{\therefore g''(-3) = 1}$$

c) $g'(x) = 0$

$g'(x) = f(x) = 0$ when $x = -1, 1$ y

$g(x)$ has a relative max @ $x=-1$ b/c $g'(x)$ changes from + to −. $g(x)$ has neither a max nor min @ $x=1$ b/c $g'(x)$ doesn't change (− to −).

d) $g''(x) = f'(x)$

$g(x)$ has a point of inflection @ $x=-2$ b/c $g''(x)$ changes from + to −, @ $x=0$ b/c $g''(x)$ changes from − to + and @ $x=1$ b/c $g''(x)$ changes from + to −.

5) $B(0) = 20$ grams

$$\frac{dB}{dt} = \frac{1}{5}(100-B)$$

a) $\frac{dB}{dt} = \frac{1}{5}(100-40) = \frac{1}{5}(60) = 12$ grams/day

$\frac{dB}{dt} = \frac{1}{5}(100-70) = \frac{1}{5}(30) = 6$ grams/day

$\therefore$ the bird is gaining weight faster when it weighs 40 grams.

b) $$\frac{d^2B}{dt^2} = \frac{1}{5}\left(0 - 1\frac{dB}{dt}\right) = -\frac{1}{5}\left(\frac{dB}{dt}\right) = -\frac{1}{5}\left(\frac{1}{5}(100-B)\right)$$

$$\frac{d^2B}{dt^2} = -\frac{1}{25}(100-B).$$

According to the graph, B is concave up for initial values of t. But, according to the second derivative, which is negative for initial values of t, the graph should start out as concave down.

c) $\dfrac{dB}{dt} = \dfrac{1}{5}(100-B)$

$$\int \frac{dB}{100-B} = \frac{1}{5}\int dt$$

$$-\ln|100-B| = \frac{1}{5}t + C$$

$$B(0)=20$$

$$-\ln|100-20| = \frac{1}{5}(0)+C$$

$$C = -\ln 80$$

$$-\ln(100-B) = \frac{1}{5}t - \ln 80$$

$$\ln(100-B) = \ln 80 - \frac{1}{5}t$$

$$100-B = e^{\ln 80 - \frac{1}{5}t} = e^{\ln 80}\cdot e^{-\frac{1}{5}t} = 80e^{-\frac{1}{5}t}$$

$$\boxed{\therefore B = 100 - 80e^{-\frac{1}{5}t}}$$

2) $y = \ln x$, $y = 5 - x$

a) Vertical cross sections (dx)

Need 2 integrals

$$R = \int_{1}^{3.693} \underbrace{\ln x}_{\text{top}} - \underbrace{0}_{\text{bottom}}\, dx + \int_{3.693}^{5} \underbrace{5 - x}_{\text{top}} - \underbrace{0}_{\text{bottom}}\, dx = 2.986$$

OR

horizontal cross sections (dy)

just needs 1 integral but, have to solve eqns. for x.

$$e^y = e^{\ln x} \qquad y = 5 - x$$

$$e^y = x \qquad x = 5 - y$$

$$R = \int_{y=0}^{y=1.307} \underbrace{(5-y)}_{\text{Right}} - \underbrace{e^y}_{\text{Left}}\, dy = 2.986$$

b) Perpendicular to the x-axis means use dx

$$V = \int \underbrace{s^2}_{\text{top - bottom}} dx = \int_{x=1}^{x=3.693} (\ln x - 0)^2 dx + \int_{x=3.693}^{x=5} (5 - x - 0)^2 dx$$

c) Since it's a horizontal line it is best to use dy.

$$\int_0^K (5-y) - e^y\, dy = \frac{1}{2}(2.986)$$

4) $f(x) = \sqrt{25 - x^2}$

a) $f'(x) = \frac{1}{2}(25 - x^2)^{-1/2}(-2x) = \dfrac{-x}{\sqrt{25 - x^2}}$

b) Point : $(-3, 4)$

$f(-3) = \sqrt{25 - (-3)^2}$
$= \sqrt{16}$
$= 4$

slope

$f'(-3) = \dfrac{-(-3)}{\sqrt{16}} = \dfrac{3}{4}$

$m = \frac{3}{4}$

$y - y_1 = m(x - x_1)$

$y - 4 = \frac{3}{4}(x + 3)$

$y = \frac{3}{4}(x + 3) + 4$

c) Defn. of continuity @ $x = c$ is

① $\lim_{x \to c} f(x)$ exists

② $f(c)$ exists

③ $\lim_{x \to c} f(x) = f(c)$

$\lim_{x \to -3} g(x)$ has to be done using one-sided limits b/c $g(x)$ is a piecewise function broken up @ $x = -3$

$\lim_{x \to -3^-} g(x) = \lim_{x \to -3^-} f(x) = f(-3) = 4$

$\lim_{x \to -3^+} g(x) = \lim_{x \to -3^+} x + 7 = -3 + 7 = 4$

$\therefore \lim_{x \to -3} g(x) = 4$ ①

② $g(-3) = f(-3) = 4$

③ $\lim_{x \to -3} g(x) = g(-3)$

$\therefore g$ is continuous @ $x = -3$.

wanted to show that you can change limits

d) $$\int_{x=0}^{x=5} x\sqrt{25-x^2}\,dx = \int_{u=25}^{u=0} x\sqrt{u}\left(\frac{du}{-2x}\right) = -\frac{1}{2}\int_{25}^{0}\sqrt{u}\,du$$

$u = 25 - x^2$

$du = -2x\,dx$

$dx = \dfrac{du}{-2x}$

$$= \frac{-\frac{1}{2}u^{3/2}}{\frac{3}{2}} = -\frac{1}{2}\cdot\frac{2}{3}u^{3/2}$$

$$= -\frac{1}{3}u^{3/2}\Big|_{25}^{0} = 0 - \left(-\frac{125}{3}\right)$$

$$= \boxed{\frac{125}{3}}$$

6) $v(t) = \cos\left(\frac{\pi}{6}t\right)$, $x(0) = -2$

a) $\cos\left(\frac{\pi}{6}t\right) = 0$

$\frac{\pi}{6}t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$

$t = 3, 9, 15, \ldots$

0		3		9		12
	1		5		10	
	+		−		+	

$\therefore$ the particle is moving to the left from $t=3$ to $t=9$ b/c $v(t)$ is −.

b) $\int_0^6 |v(t)|\,dt = \int_0^6 \left|\cos\left(\frac{\pi}{6}t\right)\right| dt$

c) $v(t) = \cos\left(\frac{\pi}{6}t\right)$

$a(t) = v'(t) = -\sin\left(\frac{\pi}{6}t\right) \cdot \frac{\pi}{6} = -\frac{\pi}{6}\sin\left(\frac{\pi}{6}t\right)$

$a(4) = -\frac{\pi}{6}\sin\left(\frac{4\pi}{6}\right) = -$

$-$ $+$

$v(4) = \cos\left(\frac{4\pi}{6}\right) = -$

$\therefore$ The speed of the particle is increasing b/c $v(4)$ and $a(4)$ are $-$.

d) $x(4) = ?$

$\int v(t) = x(t)$

$\int_0^4 v(t)\,dt = x(4) - x(0)$

$\int_0^4 \cos\left(\frac{\pi}{6}t\right)dt = x(4) - (-2)$

$\left.\frac{\sin\left(\frac{\pi}{6}t\right)}{\frac{\pi}{6}}\right|_0^4 = x(4) + 2$

$\frac{6\sin\left(\overset{\frac{2\pi}{3}}{\frac{4\pi}{6}}\right)}{\pi} - \cancel{\frac{6\sin(0)}{\pi}} \to 0$

$\frac{6\left(\frac{\sqrt{3}}{2}\right)}{\pi} = \frac{3\sqrt{3}}{\pi}$

Thus, $x(4) + 2 = \frac{3\sqrt{3}}{\pi}$

$x(4) = \boxed{\frac{3\sqrt{3}}{\pi} - 2}$