Quotient Rule WS

1) $y = \dfrac{\sqrt{x} - 4x}{2^x} = \dfrac{x^{1/2} - 4x}{2^x}$

$$\frac{dy}{dx} = \frac{\left(\frac{1}{2}x^{-1/2} - 4\right)(2^x) - (\sqrt{x} - 4x)(2^x \cdot \ln 2)}{(2^x)^2}$$

factor out

can also be written as 2^{2x} but not 2^{x^2}

$$= \frac{\cancel{2^x}\left[\left(\frac{1}{2\sqrt{x}} - 4\right) - (\sqrt{x} - 4x)(\ln 2)\right]}{(2^x)^{\cancel{2}}}$$

$$= \frac{\left(\frac{1}{2\sqrt{x}} - 4\right) - (\sqrt{x} - 4x)(\ln 2)}{2^x}$$

2) $f(x) = \dfrac{\ln x + \sin x}{3e^x}$

$$f'(x) = \frac{\left(\frac{1}{x} + \cos x\right)(3e^x) - (3e^x)(\ln x + \sin x)}{(3e^x)^2}$$

$$= \frac{\left(\frac{1}{x} + \cos x\right) - (\ln x + \sin x)}{3e^x}$$

same steps as in #1

3) $y = \dfrac{x^4 - 3x^3 + 6x^2 - 8}{x^2}$

method 1: Quotient Rule (more difficult)

$$\frac{dy}{dx} = \frac{(4x^3 - 9x^2 + 12x)(x^2) - (2x)(x^4 - 3x^3 + 6x^2 - 8)}{x^4}$$

$$= \frac{4x^5 - 9x^4 + 12x^3 - 2x^5 + 6x^4 - 12x^3 + 16x}{x^4}$$

$$= \frac{2x^5 - 3x^4 + 16x}{x^4} = \frac{2x^5}{x^4} - \frac{3x^4}{x^4} + \frac{16x}{x^4}$$

$$= 2x - 3 + \frac{16}{x^3}$$

method 2: split up numerator (easier)

$$y = \frac{x^4 - 3x^3 + 6x^2 - 8}{x^2} = \frac{x^4}{x^2} - \frac{3x^3}{x^2} + \frac{6x^2}{x^2} - \frac{8}{x^2}$$

$$y = x^2 - 3x + 6 - 8x^{-2}$$

$$\frac{dy}{dx} = 2x - 3 + 16x^{-3} = 2x - 3 + \frac{16}{x^3}$$

4) $f(x) = \frac{\csc x}{\cot x}$

method 1: Quotient Rule (more difficult)

$$f'(x) = \frac{(-\csc x \cot x)(\cot x) - (-\csc^2 x)(\csc x)}{(\cot x)^2}$$

$$f'(x) = \frac{-\csc x \cot^2 x + \csc^3 x}{\cot^2 x}$$

method 2: see Product Rule WS solutions #8

5) $y = \dfrac{9}{\ln x}$

$$\frac{dy}{dx} = \frac{(0)(\ln x) - (\frac{1}{x})(9)}{(\ln x)^2} = \frac{-\frac{9}{x}}{(\ln x)^2}$$

$$= \frac{-9}{x(\ln x)^2}$$

6) $f(x) = \dfrac{x^2 - e^x + \sin x}{\sqrt{x^3}} = \dfrac{x^2 - e^x + \sin x}{x^{3/2}}$

$$f'(x) = \frac{(2x - e^x + \cos x)(\sqrt{x^3}) - (x^2 - e^x + \sin x)(\frac{3}{2}x^{1/2})}{(\sqrt{x^3})^2}$$

$$= \frac{(2x - e^x + \cos x)(\sqrt{x^3}) - (x^2 - e^x + \sin x)(\frac{3\sqrt{x}}{2})}{x^3}$$

7) $y = -\dfrac{2x^3 - 3x}{\tan x} + \dfrac{x^4}{\ln x} - \dfrac{13}{\cos x}$

$$y = \frac{-2x^3 - 3x}{\tan x} + \frac{x^4}{\ln x} - 13\sec x$$

$$\frac{dy}{dx} = \frac{(-6x^2 - 3)(\tan x) - (\sec^2 x)(-2x^3 - 3x)}{\tan^2 x}$$

$$+ \frac{(4x^3)(\ln x) - (x^4)(\frac{1}{x})}{(\ln x)^2} - 13(\sec x \tan x)$$

not simplifying this ugly thing

8) $f(x) = \dfrac{\frac{1}{x} + \frac{3}{x^2-4}}{e^x} = \dfrac{x^{-1} + \frac{3}{x^2-4}}{e^x}$

$$f'(x) = \frac{\left[-x^{-2} + \frac{(0)(x^2-4) - (2x)(3)}{(e^x)^2}\right](e^{x})' - (e^x)\left(\frac{1}{x} + \frac{3}{x^2-4}\right)}{(e^x)^2}$$

$$= \frac{\left(\frac{-1}{x^2} + \frac{-6x}{e^{2x}}\right) - \left(\frac{1}{x} + \frac{3}{x^2-4}\right)}{e^x}$$

Note: while understanding the simplification steps is a good idea, you will never be asked to simplify such difficult problems on a quiz or on the AP test.