Quiz Prep solutions

1) $f(x) = e^{x^2}$

x-intercepts (zeros): $f(x) = 0$

$$e^{x^2} = 0$$
$$\ln e^{x^2} = \ln 0$$
$$\text{DNE}$$

$\boxed{\therefore \text{ No x-intercepts}}$

discontinuities: $f(x) = e^{x^2}$ $\boxed{\text{No discontinuities}}$

1st derivative: $f'(x) = e^{x^2}(2x) = 2xe^{x^2}$

critical points: $f'(x) = 0$ and $f'(x) = \text{undefined}$

1st $f'(x) = 0$: $2xe^{x^2} = 0$

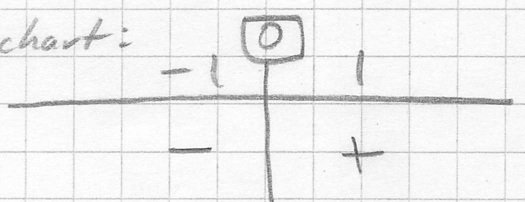
$$2x = 0 \qquad e^{x^2} = 0$$
$$x = 0 \qquad \text{No soln}$$

2nd $f'(x) = \text{undef.}$: $2xe^{x^2} = \text{undefined}$

There are no values of x that make this true

$\boxed{\therefore \text{ only critical point @ } x = 0}$

sign chart:



used for inc/dec and local extrema

$\boxed{\begin{array}{l} \therefore f(x) \text{ is dec. on } (-\infty, 0] \text{ b/c } f'(x) \text{ is neg and } f(x) \text{ is} \\ \text{inc on } [0, +\infty) \text{ b/c } f'(x) \text{ is pos.} \\ \text{Also, local min @ } x = 0 \end{array}}$

2nd derivative: $f''(x) = 2e^{x^2} + e^{x^2}(2x)(2x)$

$$f''(x) = 0$$
$$2e^{x^2}(1 + 2x^2) = 0$$
$$2e^{x^2} = 0 \qquad 1 + 2x^2 = 0$$
$$e^{x^2} = 0 \qquad \sqrt{x^2} = \sqrt{-\tfrac{1}{2}}$$
$$\text{No soln} \qquad \text{No soln}$$

∴ $f(x)$ is always concave up or always concave down. To find out which:

0 ← pick any #

+

$\boxed{\text{∴ } f(x) \text{ is concave up on } (-\infty, +\infty) \text{ b/c } f''(x) \text{ is always pos.}}$

Also, since concavity <u>never</u> changes,

No inflection pts.

2) $f(x) = x^4 - 8x$

x-intercepts: $x^4 - 8x = 0$

$x(x^3 - 8) = 0$

$x = 0 \qquad x^3 - 8 = 0$

$x^3 = 8$

$x = 2$

$\boxed{\therefore \text{ x-intercepts @ } x = 0,\ x = 2}$

discontinuities: $f(x)$ is a polynomial → $\boxed{\text{No discontinuities}}$

1st derivative: $f'(x) = 4x^3 - 8$

critical pts: $f'(x) = 0$ <u>and</u> $f'(x) = \text{undef.}$

$f'(x) = 0$: $4x^3 - 8 = 0$

$4(x^3 - 2) = 0$

$x^3 - 2 = 0$

$x = \sqrt[3]{2}$

$f'(x) = \text{undef}$: $4x^3 - 8$ is a polynomial ∴ never undef

$\boxed{\text{only critical point @ } x = \sqrt[3]{2}}$

$\sqrt[3]{2}$

0	2
−	+

∴ $f(x)$ is dec on $(-\infty, \sqrt[3]{2}]$ b/c $f'(x)$ is neg. and $f(x)$ is inc on $[\sqrt[3]{2}, +\infty)$ b/c $f'(x)$ is pos.

$\boxed{\text{Also, there is a local min. @ } x = \sqrt[3]{2}}$

2nd derivative: $f''(x) = 12x^2$

$12x^2 = 0$

$x^2 = 0$

$x = 0$

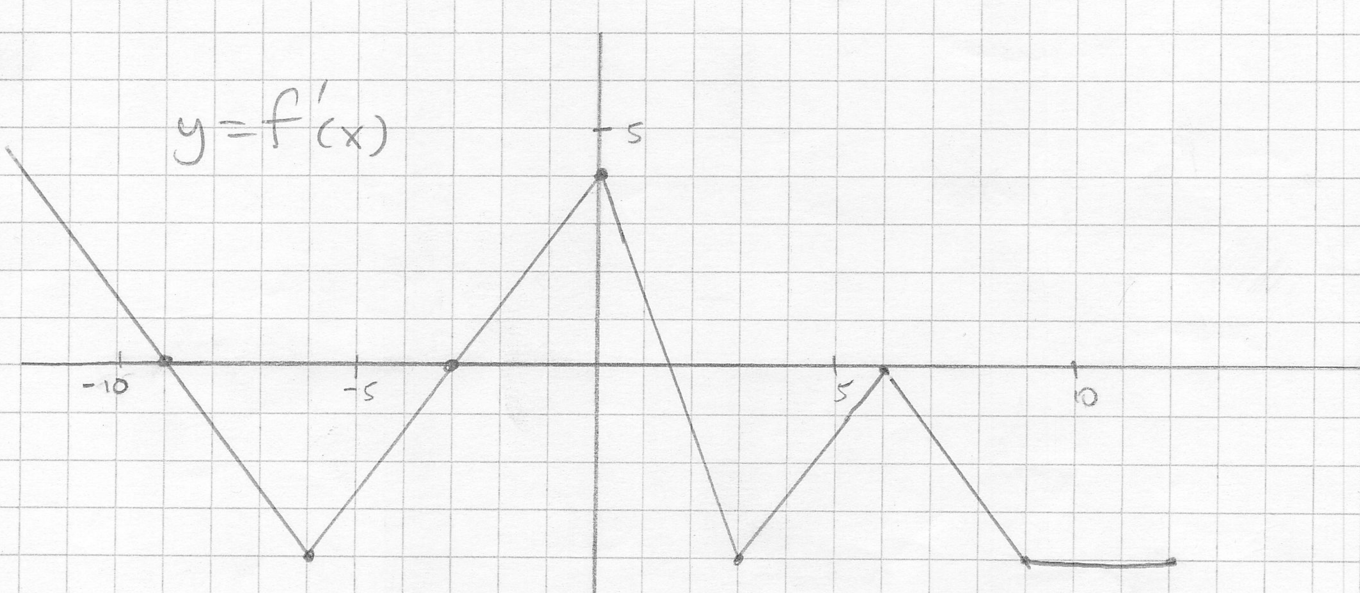
$\boxed{0}$

-1	1
+	+

∴ $f(x)$ is concave up on $(-\infty, +\infty)$ b/c $f''(x)$ is <u>always</u> positive.

Also, No inflection pt b/c no change in concavity

3)



a) $f'(0) = 4$

b) $f(-2) < f(1)$

c) $f(4) > f(10)$

d) $f''(-9) < f''(-3)$

e) $f'(4) < f''(4)$

f) critical pts @ $x = -9, -3, 1\frac{1}{2}, 6$

g) $f''(x)$ DNE @ $x = -6, 0, 3, 6, 9$

h) local max @ $x = -9, 1\frac{1}{2}$ b/c $f'(x)$ changes from pos to neg. Local min @ $x = -3$ b/c $f'(x)$ changes from neg to pos. No local max or min @ $x = 6$ b/c $f'(x)$