Proving Continuity

$f(x) = 3x^2 + x - 2$ $\qquad$ $g(x) = \begin{cases} -2x+3, & \text{if } x > -1 \\ 4-x^2, & \text{if } x \le -1 \end{cases}$

Guess: Any discont.?

Graph each function. Any discontinuities?

Show $g(x)$ in not globally continuous but is cont. on other intervals ie. $(0,4)$, $\underbrace{[1,5], (-\infty,-1)}_{\text{ask as Q's.}}$.

Note: a polynomial is cont. everywhere

What happens when we don't have the graphs to rely on? Easy to show for rational functions den $=0$

We use the defn. of continuity

A function is said to be continuous at a point $x = c$ if 3 conditions hold:

1. $f(c)$ exists $\rightarrow$ ~~substitution works~~ $f(c) = \#$
2. $\lim_{x \to c} f(x)$ exists $\rightarrow$ $= \#$, on for piecewise $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$
3. $\lim_{x \to c} f(x) = f(c)$ $\rightarrow$ substitution works in evaluating the limit

Example 1: Prove $f(x) = 3x^2 + x - 2$ is continuous at $x = 1$

Note: $g(x)$ breaks rule 2

1. $f(1) = 3(1)^2 + 1 - 2 = 2$
2. $\lim_{x \to 1} 3x^2 + x - 2 = 3(1)^2 + 1 - 2 = 2$
3. $\lim_{x \to 1} f(x) = f(1)$ $\qquad 2 = 2$ ✓ (Really only one needed)

Note: Rational functions break rule 1

Example 2 Find a value for a so that the function $f(x) = \begin{cases} x^2-1, & x<3 \\ 2ax, & x \geq 3 \end{cases}$ is cont.

Guess; where would the disc. occur?

$$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x)$$

$$\lim_{x \to 3^-} x^2-1 = \lim_{x \to 3^+} 2ax$$

$$(3)^2-1 = 2a(3)$$

$$8 = 6a$$

$$a = \frac{4}{3}$$

More examples:

1) $f(x) = \begin{cases} 2x+3, & x \leq 2 \\ ax+1, & x>2 \end{cases}$ $a=3$

2) $f(x) = \begin{cases} 4-x^2, & x<-1 \\ ax^2-1, & x \geq -1 \end{cases}$ $a=4$

3) $f(x) = \begin{cases} x^2+x+a, & x<1 \\ x^3 & x \geq 1 \end{cases}$ $a=-1$

Future HW: p. 153 #23-26, 30