

# The Product Rule WS

$$1) y = \sqrt{x} \cdot \sin x = (x^{1/2})(\sin x)$$

$$\begin{aligned} \frac{dy}{dx} &= \left(\frac{1}{2}x^{-1/2}\right)(\sin x) + (\sqrt{x})(\cos x) \\ &= \frac{\sin x}{2\sqrt{x}} + \sqrt{x} \cdot \cos x \end{aligned}$$

$$2) f(x) = \left(\frac{1}{\sqrt{x}} + x^2\right) \cot x = (x^{-1/2} + x^2)(\cot x)$$

$$\begin{aligned} f'(x) &= \left(-\frac{1}{2}x^{-3/2} + 2x\right)(\cot x) + \left(\frac{1}{\sqrt{x}} + x^2\right)(-\csc^2 x) \\ &= \left(-\frac{1}{2\sqrt{x^3}} + 2x\right)(\cot x) - \left(\frac{1}{\sqrt{x}} + x^2\right)(\csc^2 x) \end{aligned}$$

$$3) y = (4x^3 - 1)(\log_6 x)$$

$$\frac{dy}{dx} = (12x^2)(\log_6 x) + \left(\frac{1}{x \cdot \ln 6}\right)(4x^3 - 1)$$

$$4) f(x) = (2 \sec x)(\tan x)$$

$$f'(x) = (2 \sec x \tan x)(\tan x) + (2 \sec x)(\sec^2 x)$$

$$f'(x) = 2 \sec x \tan^2 x + 2 \sec^3 x$$

$$5) y = \left(4\sqrt[3]{x^2} - \frac{10}{\sqrt[4]{x}} + \frac{2}{x} - x\right)(\ln x)$$

$$\frac{dy}{dx} = \left(4x^{2/3} - 10x^{-5/4} + 2x^{-2} - x\right)(\ln x)$$

$$\begin{aligned} \frac{dy}{dx} &= \left(4\left(\frac{2}{3}x^{-1/3}\right) - 10\left(-\frac{5}{4}x^{-9/4}\right) - 2x^{-3} - 1\right)(\ln x) \\ &\quad + \left(\frac{1}{x}\right)\left(4\sqrt[3]{x^2} - \frac{10}{\sqrt[4]{x}} + \frac{2}{x} - x\right) \end{aligned}$$

$$\frac{dy}{dx} = \left( \frac{8}{3\sqrt[3]{x}} + \frac{5}{2\sqrt{x^5}} - \frac{2}{x^2} - 1 \right) (\ln x) \\ + \frac{4}{\sqrt[3]{x}} - \frac{10}{\sqrt{x^5}} + \frac{2}{x^2} - 1$$

$$6) f(x) = (4 \log x)(\cos x)$$

$$f'(x) = \left( 4 \left( \frac{1}{x \cdot \ln 10} \right) \right) (\cos x) + (-\sin x)(4 \log x) \\ = \frac{4 \cos x}{x \ln 10} - 4 \sin x \log x$$

$$7) y = \frac{6}{x^2} - 3\sqrt{x} + \frac{5}{2\sqrt{x^3}} - 10$$

$$y = 6x^{-2} - 3x^{1/2} + \frac{5}{2}x^{-3/4} - 10$$

$$\frac{dy}{dx} = -12x^{-3} - 3\left(\frac{1}{2}x^{-1/2}\right) + \frac{5}{2}\left(-\frac{3}{4}x^{-7/4}\right) \\ = -\frac{12}{x^3} - \frac{3}{2\sqrt{x}} - \frac{15}{8\sqrt{x^7}}$$

$$8) f(x) = \frac{\csc x}{\cot x} = \frac{\frac{1}{\sin x}}{\frac{\cos x}{\sin x}} = \frac{1 \cdot \cancel{\sin x}}{\cos x \cdot \cancel{\sin x}} = \frac{1}{\cos x} \\ f(x) = \sec x$$

$$f'(x) = \sec x \tan x$$

$$9) y = (x)(\ln 4x) = (x)(\ln 4 + \ln x)$$

$$\frac{dy}{dx} = (1)(\ln 4x) + (x)\left(0 + \frac{1}{x}\right) \\ = \ln 4x + 1$$

$$10) \quad f(x) = (3e)^x = (3^x)(e^x)$$
$$f'(x) = (3^x \cdot \ln 3)(e^x) + (e^x)(3^x)$$

Note: It is a good idea to try to understand the simplification steps but you will not be asked to simplify such difficult problems on your quizzes or even on the AP test.