Product and Quotient Rule WS 2

1) $y = \dfrac{\overbrace{x^3 e^x}^{u}}{\underbrace{3\ln(x)}_{v}}$

$u = x^3 e^x$
$u' = 3x^2 e^x + x^3 e^x$
$v = 3\ln(x)$
$v' = \dfrac{3}{x}$

cancelling out a coefficient

$$y' = \frac{(3x^2 e^x + x^3 e^x)(3\ln(x)) - (x^3 e^x)\left(\frac{3}{x}\right)}{3^2(\ln(x))^2}$$

$$y' = \frac{(3x^2 e^x + x^3 e^x)(\ln(x)) - x^2 e^x}{3(\ln(x))^2}$$

2) $f(x) = \dfrac{5e^{2x}}{4\sqrt{x}} = \dfrac{\overbrace{(5e^x)(e^x)}^{u}}{\underbrace{4x^{\frac{1}{2}}}_{v}}$

$u = (5e^x)(e^x)$
$u' = (5e^x)(e^x) + (5e^x)(e^x)$
$u' = 5e^{2x} + 5e^{2x}$
$u' = 10e^{2x}$
$v = 4x^{\frac{1}{2}}$
$v' = 4\left(\frac{1}{2}\right)x^{-\frac{1}{2}}$
$v' = \dfrac{2}{\sqrt{x}}$

$$f'(x) = \frac{(10e^{2x})(4\sqrt{x}) - (5e^{2x})\left(\frac{2}{\sqrt{x}}\right)}{4^2(\sqrt{x})^2} = \frac{40e^{2x}\sqrt{x} - \frac{10e^{2x}}{\sqrt{x}}}{16x}$$

$$f'(x) = \frac{20e^{2x}\sqrt{x} - \frac{5e^{2x}}{\sqrt{x}}}{8x}$$

3) $y = \dfrac{(x^4 - 12x^2 + 35)(x^3 + 14x - 3)}{6\cos(x)}$

$y = \dfrac{x^7 + 14x^5 - 3x^4 - 12x^5 - 168x^3 + 36x^2 + 35x^3 + 490x - 105}{6\cos(x)}$

$y = \dfrac{x^7 + 2x^5 - 3x^4 - 133x^3 + 36x^2 + 490x - 105}{6\cos(x)}$

$u = x^7 + 2x^5 - 3x^4 - 133x^3 + 36x^2 + 490x - 105$

$u' = 7x^6 + 10x^4 - 12x^3 - 399x^2 + 72x + 490$

$v = 6\cos(x)$

$v' = -6\sin(x)$

$y' = \dfrac{(7x^6 + 10x^4 - 12x^3 - 399x^2 + 72x + 490)(6\cos(x)) - (x^7 + 2x^5 - 3x^4 - 133x^3 + 36x^2 + 490x - 105)(-6\sin(x))}{6^2(\cos(x))^2}$

cancelling out a coefficient

$y' = \dfrac{u'(\cos(x)) + u(\sin(x))}{6(\cos(x))^2}$

4) $f(x) = \dfrac{\overbrace{(9x^4 - \frac{6}{\sqrt{x}})(10\tan(x))}^{u}}{\underbrace{9e^x}_{v}}$

$u = (9x^4 - 6x^{-\frac{1}{2}})(10\tan(x))$

$u' = (36x^3 + 3x^{-\frac{3}{2}})(10\tan(x)) + (9x^4 - \frac{6}{\sqrt{x}})(10\sec^2(x))$

$v = 9e^x$

$v' = 9e^x$

cancelling out a coeff. of e^x

$$f'(x) = \frac{u'(9e^x) - u(9e^x)}{9^2(e^x)^2} = \frac{u' - u}{9e^x}$$

5) $y = \dfrac{\overbrace{7x - e^x}^{u}}{\underbrace{\sqrt{x}(x^3-2)}_{v}}$

$u = 7x - e^x$
$u' = 7 - e^x$
$v = \sqrt{x}(x^3-2)$
$v = x^{\frac{1}{2}}(x^3-2)$
$v = x^{\frac{7}{2}} - 2x^{\frac{1}{2}}$
$v' = \frac{7}{2}x^{\frac{5}{2}} - x^{-\frac{1}{2}}$

$y' = \dfrac{u'v - uv'}{v^2}$ can't simplify anything (sorry!)

6) $f(x) = 16 + \overbrace{2^x}^{u}\overbrace{(2x^4 - 3x)}^{v}$

$u = 2^x$
$u' = 2^x \ln(2)$
$v = 2x^4 - 3x$
$v' = 8x^3 - 3$

$f'(x) = (2^x \ln(2))(2x^4 - 3x) + (2^x)(8x^3 - 3)$

7) $y = \dfrac{\overbrace{e^x + \frac{6}{x^3+1}}^{u}}{\underbrace{x}_{v}}$

$u = e^x + \frac{6}{x^3+1}$
$u' = e^x + \dfrac{(0)(x^3+1) - (6)(3x^2)}{(x^3+1)^2} = e^x - \dfrac{18x^2}{(x^3+1)^2}$

$v = x$
$v' = 1$

$$y' = \frac{\left(e^x - \frac{18x^2}{(x^3+1)^2}\right)(x) - \left(e^x + \frac{6}{x^3+1}\right)(1)}{x^2}$$

No simplifying

8) $f(x) = \dfrac{2x^4 - 5x}{e^x(x^2+7)}$

$u = 2x^4 - 5x$

$u' = 8x^3 - 5$

$v = (e^x)(x^2+7)$

$v' = (e^x)(x^2+7) + (e^x)(2x) = e^x[(x^2+7) + 2x]$

$$f'(x) = \frac{(8x^3-5)(e^x)(x^2+7) - (2x^4-5x)(e^x)(x^2+2x+7)}{(e^x)^2(x^2+7)^2}$$

cancelling out an e^x

$$f'(x) = \frac{(8x^3-5)(x^2+7) - (2x^4-5x)(x^2+2x+7)}{(e^x)(x^2+7)^2}$$

All of these problems are very complex and way more complicated than what you will see on the quiz. Practice your simplifying steps