Product Rule/Quotient Rule WS 1

1) $y = (5^x + \ln(x))(7\cos(x))$

Product Rule

$$\frac{dy}{dx} = (5^x \cdot \ln 5 + \frac{1}{x})(7\cos(x)) + (-7\sin(x))(5^x + \ln(x))$$

$$\frac{dy}{dx} = (5^x \cdot \ln 5 + \frac{1}{x})(7\cos(x)) - 7(5^x + \ln(x))(\sin(x))$$

2) $f(x) = \left(\frac{x^3}{\cot(x)}\right)(6e^x)$

$$f'(x) = \left(\frac{(3x^2\cot(x)) - (-\csc^2(x))(x^3)}{\cot^2(x)}\right)(6e^x) + (6e^x)\left(\frac{x^3}{\cot(x)}\right)$$

$$f'(x) = \left(\frac{3x^2\cot(x) + x^3\csc^2(x)}{\cot^2(x)}\right)(6e^x) + (6e^x)\left(\frac{x^3}{\cot(x)}\right)$$

3) $y = \left(\frac{\sec(x)}{\sqrt{x}}\right)\left(\frac{9}{x^3}\right) = \left(\frac{\sec(x)}{x^{1/2}}\right)(9x^{-3})$

$$\frac{dy}{dx} = \left[\frac{(\sec(x)\tan(x))(\sqrt{x}) - (\frac{1}{2}x^{-1/2})(\sec(x))}{(\sqrt{x})^2}\right]\left(\frac{9}{x^3}\right) + (-27x^{-4})\left(\frac{\sec(x)}{\sqrt{x}}\right)$$

$$\frac{dy}{dx} = \left(\frac{\sqrt{x}\sec(x)\tan(x) - \frac{\sec(x)}{2\sqrt{x}}}{x}\right)\left(\frac{9}{x^3}\right) - \frac{27\sec(x)}{x^4\sqrt{x}}$$

4) $f(x) = (3\log(x))(\csc(x))(x^2)$

$$f'(x) = \left(\frac{3}{x \cdot \ln(10)}\right)(\csc(x))(x^2) + (-\csc(x)\cot(x))(3\log(x))(x^2) + (2x)(3\log(x))(\csc(x))$$

$$f'(x) = \frac{3x^2\csc(x)}{x \cdot \ln 10} - 3x^2\csc(x)\cot(x)\log(x) + 3(2x)\log(x)\csc(x)$$

$$f'(x) = \frac{3x\csc(x)}{\ln 10} - 3x^2\csc(x)\cot(x)\log(x) + 6x\log(x)\csc(x)$$

5) $$y = \frac{\frac{1}{x} - \frac{3}{x^2}}{x^4} = \frac{\frac{x}{x^2} - \frac{3}{x^2}}{x^4} = \frac{\frac{x-3}{x^2}}{x^4} = \frac{x-3}{x^6} = \frac{x}{x^6} - \frac{3}{x^6}$$

$$y = \frac{1}{x^5} - \frac{3}{x^6} = x^{-5} - 3x^{-6}$$

$$\frac{dy}{dx} = -5x^{-6} + 18x^{-7} = -\frac{5}{x^6} + \frac{18}{x^7}$$

6) $$f(x) = \frac{6}{\sin(x)} - \frac{x}{\tan(x)} + \frac{10}{\sec(x)}$$

$$f(x) = 6\csc(x) - \frac{x}{\tan(x)} + 10\cos(x)$$

* could have also re-written to be $x \cdot \cot(x)$

$$f'(x) = 6(-\csc(x)\cot(x)) - \left[\frac{1(\tan(x)) - (\sec^2(x))(x)}{\tan^2(x)}\right] + 10(-\sin x)$$

$$f'(x) = -6\csc(x)\cot(x) - \left[\frac{\tan(x) - x\sec^2(x)}{\tan^2(x)}\right] - 10\sin(x)$$

7) $y = \dfrac{4x \ln(x)}{e^x}$

$$\frac{dy}{dx} = \frac{\left[(4)(\ln(x)) + \left(\frac{1}{x}\right)(4x)\right](e^x) - (e^x)(4x\ln(x))}{(e^x)^2}$$

$$\frac{dy}{dx} = \frac{(4\ln(x) + 4) - 4x\ln(x)}{e^x}$$

8) $$f(x) = \left(\frac{\frac{1}{x} + \frac{2}{\cos(x)}}{x^2}\right) - \left(\frac{xe^x}{\sec(x)}\right)$$

$$f(x) = \left(\frac{x^{-1} + 2\sec(x)}{x^2}\right) - \left(xe^x\cos(x)\right)$$

$$f'(x) = \frac{\left(-1x^{-2} + 2\sec(x)\tan(x)\right)(x^2) - (2x)\left(\frac{1}{x} + 2\sec(x)\right)}{(x^2)^2 \rightarrow x^4}$$

$$- \left[(1)(e^x)(\cos(x)) + (e^x)(x)(\cos(x)) + (-\sin(x))(x)(e^x)\right]$$

$$f'(x) = \frac{\left(-\frac{1}{x^2} + 2\sec(x)\tan(x)\right)(x) - 2\left(\frac{1}{x} + 2\sec(x)\right)}{x^3}$$

$$- \left[e^x\cos(x) + xe^x\cos(x) - xe^x\sin(x)\right]$$