Answers to Problem Set 8 WS

1) $y = 3x^2 - x \quad @ x = 1$

$\frac{dy}{dx} = 6x - 1 \qquad \left.\frac{dy}{dx}\right|_{x=1} = 6(1) - 1 = 5$

$y = 3(1)^2 - 1 = 2$

$(1, 2)$

$m_{tan} = 5$

$\boxed{y - 2 = 5(x - 1)}$

2) $y = x^3 - 3x \quad @ x = 3$

$\frac{dy}{dx} = 3x^2 - 3 \qquad \left.\frac{dy}{dx}\right|_{x=3} = 3(3)^2 - 3 = 24$

$y = 3^3 - 3(3) = 18$

$(3, 18)$

$m_{tan} = 24$

$\boxed{y - 18 = 24(x - 3)}$

3) $y = \sqrt{8x} = (8x)^{1/2} \quad @ x = 2$

$\frac{dy}{dx} = \frac{1}{2}(8x)^{-1/2}(8) = \frac{4}{\sqrt{8x}} \qquad \left.\frac{dy}{dx}\right|_{x=2} = \frac{4}{\sqrt{16}} = 1$

$y = \sqrt{8(2)} = 4$

$(2, 4)$

$m_{tan} = 1$ so, $m_{\perp} = -1$

$\boxed{y - 4 = -1(x - 2)}$

4) $y = \frac{1}{\sqrt{x^2+7}} = (x^2+7)^{-1/2}$ @ $x = 3$

$$\frac{dy}{dx} = -\frac{1}{2}(x^2+7)^{-3/2}(2x) = \frac{-x}{\sqrt{(x^2+7)^3}}$$

$$\left.\frac{dy}{dx}\right|_{x=3} = \frac{-3}{\sqrt{(3^2+7)^3}} = \frac{-3}{\sqrt{16^3}} = \frac{-3}{4^3} = \frac{-3}{64}$$

$$y = \frac{1}{\sqrt{3^2+7}} = \frac{1}{\sqrt{16}} = \frac{1}{4} \qquad (3, \tfrac{1}{4})$$

$m_{tan} = -\frac{3}{64}$

$$\boxed{y - \tfrac{1}{4} = -\tfrac{3}{64}(x-3)}$$

5) $y = \frac{x+3}{x-3}$ @ $x = 4$

$$\frac{dy}{dx} = \frac{1(x-3) - 1(x+3)}{(x-3)^2} = \frac{x-3-x-3}{(x-3)^2} = \frac{-6}{(x-3)^2}$$

$$\left.\frac{dy}{dx}\right|_{x=4} = \frac{-6}{(4-3)^2} = -6$$

$m_{tan} = -6$ so, $m_{\perp} = \frac{1}{6}$

$y = \frac{4+3}{4-3} = 7 \qquad (4, 7)$

$$\boxed{y - 7 = \tfrac{1}{6}(x-4)}$$

6) $y = 4 - 3x - x^2$ @ $(0,4)$

$\frac{dy}{dx} = -3 - 2x$ $\quad \left.\frac{dy}{dx}\right|_{x=0} = -3 - 2(0) = -3$

$m_{tan} = -3$

$(0,4)$

$\boxed{y - 4 = -3x}$

7) $y = 2x^3 - 3x^2 - 12x + 20$ @ $x = 2$

$\frac{dy}{dx} = 6x^2 - 6x - 12$ $\quad \left.\frac{dy}{dx}\right|_{x=2} = 6(2)^2 - 6(2) - 12 = 0$

$m_{tan} = 0$ which means that the tangent line @ $x = 2$ is a horizontal line.

$y = 2(2)^3 - 3(2)^2 - 12(2) + 20 = 16 - 12 - 24 + 20 = 0$

$m_{tan} = 0$

$(2,0)$

$y - 0 = 0(x - 2)$

$\boxed{y = 0}$

8) $y = \frac{x^2+4}{x-6}$ @ $x = 5$

$y = \frac{5^2+4}{5-6} = \frac{29}{-1} = -29$

$(5, -29)$

$m_{tan} = \frac{dy}{dx} = \frac{2x(x-6) - 1(x^2+4)}{(x-6)^2} = \frac{2x^2-12x-x^2-4}{(x-6)^2}$

$\frac{dy}{dx} = \frac{x^2-12x-4}{(x-6)^2}$ $\left.\frac{dy}{dx}\right|_{x=5}$ $\frac{(5)^2-12(5)-4}{(5-6)^2} = \frac{25-60-4}{(-1)^2} = -39$

$m_{tan} = -39$

$\boxed{y+29 = -39(x-5)}$

9) $y = \sqrt{x^3-15} = (x^3-15)^{1/2}$ @ $(4, 7)$

$m_{tan} = \frac{dy}{dx} = \frac{1}{2}(x^3-15)^{-1/2}(3x^2) = \frac{3x^2}{2\sqrt{x^3-15}}$

$\left.\frac{dy}{dx}\right|_{x=4} = \frac{3(4)^2}{2\sqrt{4^3-15}} = \frac{48}{2(7)} = \frac{24}{7}$

$\boxed{y-7 = \frac{24}{7}(x-4)}$

10) $y = (x^2+4x+4)^2$ @ $x = -2$

$y = [(-2)^2+4(-2)+4]^2 = (4-8+4)^2 = 0$ $(-2, 0)$

$m_{tan} = \frac{dy}{dx} = 2(x^2+4x+4)^1(2x+4)$

$\left.\frac{dy}{dx}\right|_{x=-2} = 2[(-2)^2+4(-2)+4][2(-2)+4] = 0 = m_{tan}$

$\boxed{y = 0}$

12) $y = \frac{3x+5}{x-1}$ @ $x = 3$

$y = \frac{3(3)+5}{3-1} = \frac{9+5}{2} = \frac{14}{2} = 7$

$(3, 7)$

$\frac{dy}{dx} = \frac{3(x-1) - 1(3x+5)}{(x-1)^2} = \frac{3x-3-3x-5}{(x-1)^2} = \frac{-8}{(x-1)^2}$

$\frac{dy}{dx}\Big|_{x=3} = \frac{-8}{(3-1)^2} = \frac{-8}{4} = -2$

$m_{tan} = -2$ so, $m_{\perp} = \frac{1}{2}$

$\boxed{y - 7 = \frac{1}{2}(x-3)}$

13) $y = (x-9)^2$ parallel to y-axis

slope of y-axis?

y-axis is a vertical line and all vertical lines have an undefined slope

normal

$m_{tan} = \frac{dy}{dx} = 2(x-9) = 2x-18$

$m_{\perp} = -\frac{1}{2x-18}$ ← parallel lines have same slope

$-\frac{1}{2x-18}$ = undefined

Q: how does something become undefined?

A: dividing by zero

$2x - 18 = 0$

$\boxed{x = 9}$

14) $y = 8 - 3x - x^2$ parallel to x-axis

tangent

x-axis is a horizontal line

horiz lines have $m = 0$

$m_{tan} = -3 - 2x$ ← parallel lines have the same slope

$-3 - 2x = 0$

$-2x = 3$

$x = -\frac{3}{2}$ doesn't just want x-values, want coordinates (point)

Q: how do we find a point on the graph?

A: plug into original function

$y = 8 - 3(-\frac{3}{2}) - (-\frac{3}{2})^2 = 8 + \frac{9}{2} - \frac{9}{4} = 8 + \frac{9}{4}$

$y = \frac{41}{4}$

$\boxed{(-\frac{3}{2}, \frac{41}{4})}$

15) $y = x^2 + ax + b$ $y = cx + x^2$

tangent @ $(-1, 0)$

$m_{tan} = \frac{dy}{dx} = 2x + a$ $m_{tan} = c + 2x$

common tangent means $2x + a = c + 2x$

$2x + a = c + 2x$

$-2x \quad -2x$

$a = c$

plug in point

$0 = (-1)^2 + a(-1) + b$ $0 = c(-1) + (-1)^2$

$0 = 1 - a + b$ $0 = -c + 1$

$\boxed{c = 1}$

$a = c \Rightarrow \boxed{a = 1} \Rightarrow 0 = 1 - a - b \Rightarrow 0 = 1 - 1 - b \Rightarrow \boxed{b = 0}$