Problem Set 12 WS solutions

1) Area of a circle

$A = \pi r^2$

$\frac{dA}{dt} = \pi(2r)\frac{dr}{dt}$

solve for $\frac{dA}{dt}$.

$* \frac{dA}{dt} = \pi(2)(r)\frac{dr}{dt}$ ← need to solve for r and $\frac{dr}{dt}$ with other formulas.

solve for r using $C = 2\pi r$

$100\pi = 2\pi r$

$r = 50$

solve for $\frac{dr}{dt}$ by taking the derivative of $C = 2\pi r$

$\frac{dC}{dt} = 2\pi\frac{dr}{dt}$

$40 = 2\pi\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{20}{\pi}$

$\frac{dA}{dt} = \pi(2)(50)\left(\frac{20}{\pi}\right) = 2000$

Area is increasing at a rate of $2000\ ft^2/sec$

2) $27\ \frac{in^3}{sec}$ → Volume → Volume of a sphere

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = \frac{4}{3}\pi(3r^2)\frac{dr}{dt}$

solve for $\frac{dr}{dt}$.

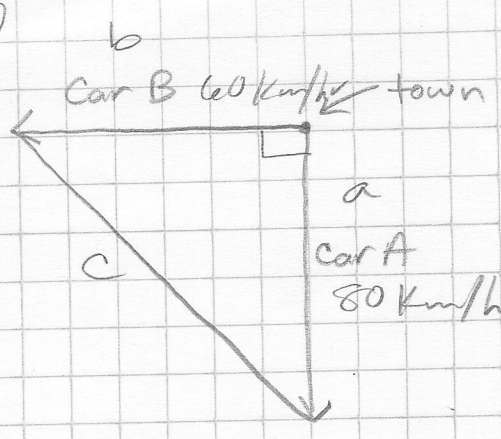
$27\pi = \frac{4}{3}\pi(3)(3)^2\frac{dr}{dt}$

$27\pi = 36\pi\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{3}{4}\ \frac{in}{sec}$

∴ radius is increasing at a rate of $\frac{3}{4}\ \frac{in}{sec}$

3)



Pythagorean Th^m

$a^2+b^2=c^2$

a, b, and c are changing with time

$2a\frac{da}{dt}+2b\frac{db}{dt}=2c\frac{dc}{dt}$ ✱

Problem wants us to solve for $\frac{dc}{dt}$

how do we find a, b, and c.

Use $D=R\cdot T$ to find a and b.

Find a: a is the distance car A travelled in 3 hrs going 80 Km/hr.

$D=(80)(3)=240 \quad \therefore a=240$

Find b: b is the distance car B travelled in 3 hrs going 60 Km/hr

$D=(60)(3)=180 \quad \therefore b=180$

Find c using Pythagorean Th^m: $a^2+b^2=c^2$

$240^2+180^2=c^2$

$c=300$

Plug a, b and c back into deriv. form.

$2(240)(80)+2(180)(60)=2(300)\frac{dc}{dt}$

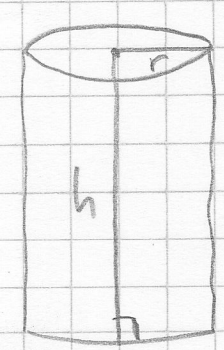
$38400+21600=600\frac{dc}{dt}$

$60000=600\frac{dc}{dt}$

$\frac{dc}{dt}=100$ Km/hr

$\therefore$ distance between 2 cars is increasing at a rate of 100 Km/hr

4)



$108\pi\ \frac{m^3}{sec}$ → Volume → Volume of a cylinder

$V=\pi r^2 h$

V and h change with time, not r

$\frac{dV}{dt}=\pi r^2\frac{dh}{dt}$ (r^2 is a coeff.)

Wants us to solve for $\frac{dh}{dt}$.

$$108\pi = \pi (6)^2 \frac{dh}{dt}$$

$$\frac{dh}{dt} = 3 \text{ m/sec}$$

∴ height is increasing at a rate of 3 m/sec