Product / Quotient Rule WS 3

1) $f(x) = \dfrac{\overbrace{\tan(x)}^{u}}{\underbrace{4x^2}_{v}}$

S.W.

$u = \tan(x)$
$u' = \sec^2(x)$
$v = 4x^2$
$v' = 8x$

$f'(x) = \dfrac{(\sec^2(x))(4x^2) - (\tan(x))(8x)}{16x^4}$

$f'(x) = \dfrac{x\sec^2(x) - 2\tan(x)}{4x^3}$

2) $f(x) = \dfrac{\overbrace{3x^2\sin(x)}^{u}}{\underbrace{6e^x}_{v}}$

S.W.

$u = 3x^2\sin(x)$
$u' = 6x\sin(x) + 3x^2\cos(x)$
$v = 6e^x$
$v' = 6e^x$

Note: you can simplify the 3 and the 6 before you take the derivative. Would be easier if you did

$f'(x) = \dfrac{(6x\sin(x) + 3x^2\cos(x))(6e^x) - (3x^2\sin(x))(6e^x)}{6^2(e^x)^2}$ 36

$f'(x) = \dfrac{3x(2\sin(x) + x\cos(x))(6e^x) - 18x^2e^x\sin(x)}{36e^{2x}}$

$f'(x) = \dfrac{\cancel{18}^{1}x\cancel{e^x}^{1}(2\sin(x) + x\cos(x)) - \cancel{18}^{1}x^2\cancel{e^x}^{1}\sin(x)}{\cancel{36}^{2}\cancel{e^{2x}}^{e^x}}$

$f'(x) = \dfrac{x(2\sin(x) + x\cos(x)) - x^2\sin(x)}{2e^x}$

3) $f(x) = (\overbrace{4x}^{u})(\overbrace{2^x}^{v}) - 2^x$

$f'(x) = (4)(2^x) + (4x)(2^x\ln(2)) - 2^x\ln(2)$

$f'(x) = 2^x(4 + 4x\ln(2) - \ln(2))$

S.W

$u = 4x$
$u' = 4$
$v = 2^x$
$v' = 2^x\ln(2)$

4) $f(x) = \dfrac{\overbrace{2x^2 - x + 5}^{u}}{\underbrace{3x+2}_{v}}$

S.W

$u = 2x^2 - x + 5$

$u' = 4x - 1$

$v = 3x + 2$

$v' = 3$

$$f'(x) = \frac{(4x-1)(3x+2) - (2x^2-x+5)(3)}{(3x+2)^2}$$

$$f'(x) = \frac{12x^2 + 8x - 3x - 2 - (6x^2 - 3x + 15)}{(3x+2)(3x+2)}$$

$$f'(x) = \frac{12x^2 + 8x - 3x - 2 - 6x^2 + 3x - 15}{9x^2 + 6x + 6x + 4}$$

$$f'(x) = \frac{6x^2 + 8x - 17}{9x^2 + 12x + 4}$$

5) $f(x) = \dfrac{\cot(x)}{1 + \cot(x)}$

S.W.

$u = \cot(x)$

$u' = -\csc^2(x)$

$v = 1 + \cot(x)$

$v' = -\csc^2(x)$

$$f'(x) = \frac{(-\csc^2(x))(1+\cot(x)) - (\cot(x))(-\csc^2(x))}{(1+\cot(x))^2}$$

$$f'(x) = \frac{-\csc^2(x) - \csc^2(x)\cot(x) + \csc^2(x)\cot(x)}{(1+\cot(x))^2}$$

$$f'(x) = \frac{-\csc^2(x)}{(1+\cot(x))^2}$$

6) $f(x) = \dfrac{x+1}{x-1}$

S.W.
$u = x+1$
$u' = 1$
$v = x-1$
$v' = 1$

$$f'(x) = \frac{(1)(x-1) - (x+1)(1)}{(x-1)^2}$$

$$f'(x) = \frac{x-1-x-1}{(x-1)^2} = \frac{-2}{(x-1)^2}$$

7) $f(x) = \dfrac{(4x^2-16)(4x^2+16)}{(x+5)(x-5)} = \dfrac{16x^4+64x^2-64x^2-256}{x^2+5x-5x-25}$

$$f(x) = \frac{16x^4-256}{x^2-25}$$

$u = 16x^4 - 256$
$u' = 64x^3$
$v = x^2-25$
$v' = 2x$

$$f'(x) = \frac{(64x^3)(x^2-25) - (16x^4-256)(2x)}{(x^2-25)^2}$$

$$f'(x) = \frac{64x^5 - 1600x^3 - (32x^5 - 512x)}{(x^2-25)(x^2-25)}$$

$$f'(x) = \frac{32x^5 - 1600x^3 + 512x}{x^4 - 25x^2 - 25x^2 + 625}$$

$$f'(x) = \frac{32x^5 - 1600x^3 + 512x}{x^4 - 50x^2 + 625}$$

8) $f(x) = \underbrace{\sqrt[5]{x}}_{u}\ \underbrace{(\cot(x))}_{v}\underbrace{(e^x)}_{w}$

$u = x^{\frac{1}{5}}$

$u' = \frac{1}{5}x^{-\frac{4}{5}} = \frac{1}{5\sqrt[5]{x^4}}$

$v = \cot(x)$

$v' = -\csc^2(x)$

$w = e^x$

$w' = e^x$

$$f'(x) = \left(\frac{1}{5\sqrt[5]{x^4}}\right)(\cot(x))(e^x) + (\sqrt[5]{x})(-\csc^2(x))(e^x) + (\sqrt[5]{x})(\cot(x))(e^x)$$

$$f'(x) = \left(\frac{1}{5\sqrt[5]{x^4}}\right)(\cot(x))(e^x) - (\sqrt[5]{x})(\csc^2(x))(e^x) + (\sqrt[5]{x})(\cot(x))(e^x)$$

9) $f(x) = x^8 \sec(x)$

$u = x^8$

$u' = 8x^7$

$v = \sec(x)$

$v' = \sec(x)\tan(x)$

$$f'(x) = (8x^7)(\sec(x)) + (x^8)(\sec(x)\tan(x))$$

10) $f(x) = \frac{16}{\sin(x)} = 16\csc(x)$

$$f'(x) = 16(-\csc(x)\cot(x)) = -16\csc(x)\cot(x)$$