# Practice Quiz 17

2011 #1

1) a) Speed increasing → speeding up

For a particle to be speeding up the velocity and acceleration **must** have the same sign.

$V(5.5) = 2\sin(e^{5.5/4}) + 1 = {}^{-}0.453$

$a(5.5) = \frac{1}{2}e^{5.5/4}\cos(e^{5.5/4}) = -1.359$

Since the velocity and acceleration are both negative The particle is speeding up @ $t = 5.5$.

b) Average value formula: Average value of $f(x) = \frac{1}{b-a}\int_a^b f(x)\,dx$

So, average velocity $= \frac{1}{b-a}\int_a^b v(t)\,dt$

$= \frac{1}{6-0}\int_0^6 2\sin(e^{t/4}) + 1 = 1.949$

c) Distance $= \int_0^6 |v(t)|\,dt = \int_0^6 |2\sin(e^{t/4}) + 1| = 12.573$

d) In order for the particle to change direction it must first stop. ∴ set $V(t) = 0$

$2\sin(e^{t/4}) + 1 = 0$

$t = 5.196$ (from "zero" function on calculator)

particle changes direction b/c velocity changes from + to −.

$\int v(t) = X(t) \Longrightarrow \int_0^{5.196} v(t) = X(5.196) - X(0)$

$$X(5.196) = \int_0^{5.196} v(t)\,dt + X(0) = 12.135 + 2 = 14.135$$

2011 #3

3. $f(x) = 8x^3$, $g(x) = \sin(\pi x)$

a) Eqn of a line requires a point and a slope.

point: $x = \frac{1}{2}$, $y = 8x^3$

$$y = 8\left(\tfrac{1}{2}\right)^3 = 8\left(\tfrac{1}{8}\right) = 1 \qquad \text{point } \left(\tfrac{1}{2}, 1\right)$$

slope: slope of tangent line means to use 1st derivative

$$f'(x) = 24x^2$$

$$m_{tan} @ x = \tfrac{1}{2} = f'\left(\tfrac{1}{2}\right) = 24\left(\tfrac{1}{2}\right)^2 = 24\left(\tfrac{1}{4}\right) = 6$$

eqn of tangent line: $y - y_1 = m(x - x_1)$

$$y - 1 = 6\left(x - \tfrac{1}{2}\right)$$

b) 1st $\rightarrow$ choose your rectangle

I choose a vertical rectangle ($dx$)

setup: $\int \quad dx \Longrightarrow \int_0^{\frac{1}{2}} \text{top} - \text{bottom}\,dx$

$$= \int_0^{\frac{1}{2}} \sin(\pi x) - 8x^3\,dx$$

$$= \frac{-\cos(\pi x)}{\pi} - 2x^4 \Big|_0^{\frac{1}{2}} = \left(0 - \tfrac{1}{8}\right) - \left(-\tfrac{1}{\pi} - 0\right)$$

$$= \frac{1}{\pi} - \frac{1}{8}$$

c) I choose a verticle rectangle again ($dx$) $\Rightarrow$ Washer method

$$V = \pi \int_0^{\frac{1}{2}} R^2 - r^2\,dx = \int_0^{\frac{1}{2}} (1 - 8x^3)^2 - (1 - \sin\pi x)^2\,dx$$