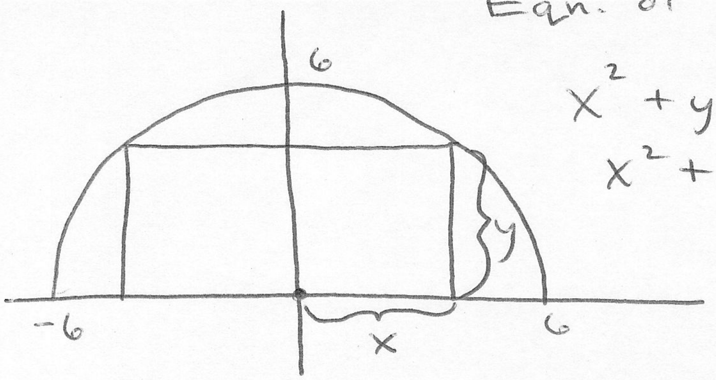
Practice Quiz 1/27

1)

Eqn. of a circle: $x^2 + y^2 = r^2$



$$x^2 + y^2 = 6^2$$
$$x^2 + y^2 = 36$$
$$y^2 = 36 - x^2$$
$$y = \pm\sqrt{36 - x^2}$$
$$y = \sqrt{36 - x^2}$$

$$A = l \cdot w$$
$$A = (2x)(y)$$
$$A = (2x)\left(\sqrt{36 - x^2}\right) = (2x)(36 - x^2)^{\frac{1}{2}}$$

$$\frac{dA}{dx} = (2)\left(\sqrt{36 - x^2}\right) + (2x)\left(\frac{-\cancel{2}x}{\cancel{2}\sqrt{36 - x^2}}\right)$$

$$\frac{dA}{dx} = \frac{2\sqrt{36 - x^2}}{1} + \frac{-2x^2}{\sqrt{36 - x^2}}$$

$$\frac{dA}{dx} = \frac{2(36 - x^2)}{\sqrt{36 - x^2}} + \frac{-2x^2}{\sqrt{36 - x^2}}$$

$$\frac{dA}{dx} = \frac{72 - 2x^2 - 2x^2}{\sqrt{36 - x^2}} = \frac{72 - 4x^2}{\sqrt{36 - x^2}}$$

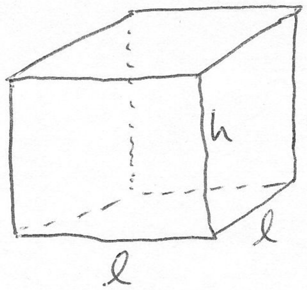
critical pts: $72 - 4x^2 = 0$ $\quad$ $\sqrt{36 - x^2} = 0$ $\quad$ doesn't make sense

$x = \pm\sqrt{18}$ $\quad$ $x = \pm 6$

Only value that . $x = \sqrt{18}$

the area of the largest rect. is

2)



~~least~~ least amount of glass means min. surface area

$V = 108\ ft^3$

$108 = l \cdot w \cdot h$

$108 = l \cdot l \cdot h$

$108 = l^2 h$

$h = \frac{108}{l^2}$

$S = 2lh + 2lh + l^2$

$S = 4lh + l^2$

$S = 4l\left(\frac{108}{l^2}\right) + l^2$

$S = 432 l^{-1} + l^2$

$S' = \frac{-432}{l^2} + 2l$

$S' = \frac{-432}{l^2} + \frac{2l^3}{l^2}$

$S' = \frac{2l^3 - 432}{l^2}$

critical pts:

$2l^3 - 432 = 0 \quad , \quad l^2 = 0$

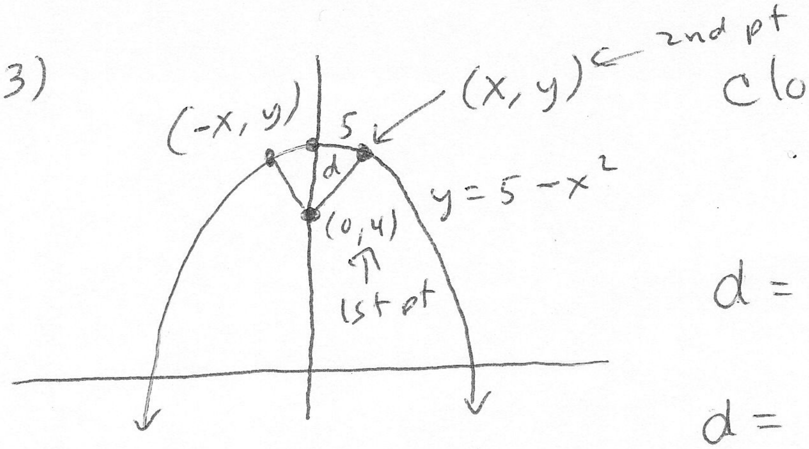
$l^3 = 216 \qquad \cancel{l = 0}$

$l = 6$

$h = \frac{108}{6^2} = 3$

$\boxed{\text{the dimensions that min. the amount of glass are 6 ft by 6 ft by 3 ft}}$

3)



Closest point means mini. dist. formula between 2 pts.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x-0)^2 + (y-4)^2}$$

$$d = \sqrt{x^2 + (5 - x^2 - 4)^2}$$

$$d = \sqrt{x^2 + (1-x^2)^2}$$

$$d = \sqrt{x^2 + 1 - 2x^2 + x^4}$$

$$d = (x^4 - x^2 + 1)^{\frac{1}{2}}$$

Side work

$(1-x^2)^2$

$(1-x^2)(1-x^2)$

$1 - x^2 - x^2 + x^4$

$1 - 2x^2 + x^4$

$$d' = \frac{1(4x^3 - 2x)}{2\sqrt{x^4 - x^2 + 1}}$$

critical pts: $4x^3 - 2x = 0$, $2\sqrt{x^4 - x^2 + 1} = 0$

$2x(2x^2 - 1) = 0$ $\qquad \sqrt{x^4 - x^2 + 1} = 0$

$2x = 0 \quad 2x^2 - 1 = 0$ $\qquad x^4 - x^2 + 1 = 0$

$x = 0 \quad x^2 = \frac{1}{2}$ $\qquad$ No soln.

$x = \pm\sqrt{\frac{1}{2}}$

$\boxed{x = 0}$ $\quad Y_1 = (x^4 - x^2 + 1)^{\frac{1}{2}}$

$Y_1(0) = 1$

$\boxed{x = \sqrt{\frac{1}{2}}}$ $\quad Y_1(\sqrt{\frac{1}{2}}) = .866$

$\boxed{x = -\sqrt{\frac{1}{2}}}$ $\quad Y_1(-\sqrt{\frac{1}{2}}) = .866$

} Winners

$\boxed{\text{the closest points are } (\sqrt{\tfrac{1}{2}}, 4\tfrac{1}{2}) \text{ and } (-\sqrt{\tfrac{1}{2}}, 4\tfrac{1}{2})}$

4)

$$\frac{dr}{dt} = 5 \, \frac{in}{hr}$$

$$r = 3 \text{ in}$$

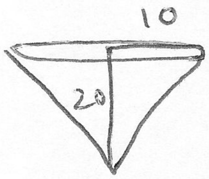
$$A = \pi r^2$$

$$\frac{dA}{dt} = \pi (2r) \frac{dr}{dt}$$

$$\frac{dA}{dt} = 2\pi (3)(5) = 30\pi \, \frac{in^2}{hr}$$

$\boxed{\text{The area is dec. @ a rate of } 30\pi \, \frac{in^2}{hr}.}$

5)



water level means height

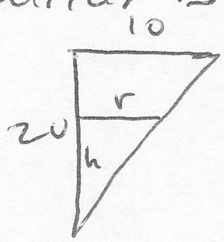
changes

$$V = \frac{1}{3}\pi r^2 h$$

$$\frac{dh}{dt} = 3 \text{ cm/sec}$$

$$h_{H_2O} = 6 \text{cm}$$

Similar $\triangle$'s



$$\frac{10}{20} = \frac{r}{h}$$

$$10h = 20r$$

$$r = \frac{10h}{20} = \frac{h}{2}$$

So,

$$V = \frac{1}{3}\pi\left(\frac{h}{2}\right)^2 (h) = \frac{\pi}{12} h^3$$

$$\frac{dV}{dt} = \frac{\pi}{12}(3h^2)\frac{dh}{dt}$$

$$\frac{dV}{dt} = \frac{\pi}{4}(6)^2(3) = 27\pi \, \frac{cm^3}{sec}$$

$\boxed{\text{the volume of the water dec @ a rate of } 27\pi \, \frac{cm^3}{sec}}$