Practice Quiz 3 WS

1) $\lim_{x\to 1} \frac{x-1}{x^3-1} = \frac{0}{0}$ indeterminate form

$\lim_{x\to 1} \frac{\cancel{(x-1)}}{\cancel{(x-1)}(x^2+x+1)} = \frac{1}{1^2+1+1} = \frac{1}{3}$

$\therefore$ removable discont. @ $x=1$

2) $\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} = \frac{\sqrt{1}-1}{0} = \frac{0}{0}$ indeterminate form

$\lim_{x\to 0} \frac{\sqrt{x+1}-1}{x} \cdot \frac{(\sqrt{x+1}+1)}{(\sqrt{x+1}+1)} = \frac{(\sqrt{x+1})^2-1}{x(\sqrt{x+1}+1)} = \frac{x+1-1}{x(\sqrt{x+1}+1)}$

$= \frac{\cancel{x}}{\cancel{x}(\sqrt{x+1}+1)} = \frac{1}{\sqrt{x+1}+1} = \frac{1}{1+1} = \frac{1}{2}$

$\therefore$ removable disc. @ $x=0$

3) $\lim_{x\to 0} \frac{\sqrt{x+1}+1}{x} = \frac{1+1}{0} = \frac{2}{0}$ Not an indet. form

$\lim_{x\to 0^-} \frac{\sqrt{x+1}+1}{x} = \frac{2}{0^-} = -\infty$, $\lim_{x\to 0^+} \frac{\sqrt{x+1}+1}{x} = \frac{2}{0^+} = +\infty$

$\therefore \lim_{x\to 0} \frac{\sqrt{x+1}+1}{x}$ DNE. Also, infinite disc @ $x=0$

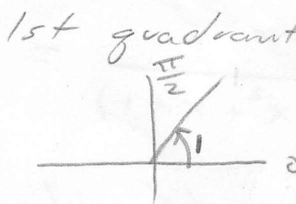
4) $\lim_{x\to 0} \frac{\sin 3x}{x} = \frac{0}{0}$ indet. form

$\lim_{x\to 0} \frac{\sin 3x}{x} \cdot \frac{(3)}{(3)} = \lim_{x\to 0} \frac{3\sin 3x}{3x} = \lim_{x\to 0} 3 \cdot \lim_{x\to 0} \frac{\sin 3x}{3x}$

$= 3 \cdot 1 = \boxed{3}$ $\therefore$ removable disc. @ $x=0$

5) $\lim_{x \to 1} \frac{\cos x}{(x-1)^2} = \frac{\#}{0}$ ← this $\# \neq 0$ but is it positive or negative?

To find out w/o a calculator, know that $0 < 1 < \frac{\pi}{2} \approx 1.57$. $\therefore$ $x = 1$ is in the 1st quadrant of the unit circle,



and all trig values are positive in the 1st quad.

Thus, $\lim_{x \to 1} \frac{\cos x}{(x-1)^2} = \frac{\text{positive } \# \neq 0}{0}$

$\lim_{x \to 1^-} \frac{\cos x}{(x-1)^2} = \frac{\text{pos } \#}{0^+} = +\infty$ and $\lim_{x \to 1^+} \frac{\cos x}{(x-1)^2} = \frac{\text{pos } \#}{0^+} = +\infty$

$\therefore$ $\lim_{x \to 1} \frac{\cos x}{(x-1)^2} = +\infty$ and infinite disc @ $x = 1$

6) $\lim_{x \to 1} \frac{\sin x}{x-1} = \frac{\text{pos } \# \neq 0}{0}$ ← for same reason as #5 above

$\lim_{x \to 1^-} \frac{\sin x}{x-1} = \frac{\text{pos } \#}{0^-} = -\infty$, $\lim_{x \to 1^+} \frac{\sin x}{x-1} = \frac{\text{pos } \#}{0^+} = +\infty$

$\therefore$ $\lim_{x \to 1} \frac{\sin x}{x-1}$ DNE and infinite disc @ $x = 1$

7) $\lim_{x \to 2} f(x)$ exists $\overset{\text{implies}}{\Longrightarrow}$ $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$\lim_{x \to 2} f(x) = f(2) \Longrightarrow \lim_{x \to 2^-} f(x) = f(2)$ and $\lim_{x \to 2^+} f(x) = f(2)$

$\therefore$ $\lim_{x \to 2^-} px^2 - 3qx - 18 = f(2)$, $\lim_{x \to 2^+} 2px + 5q - 7 = f(2)$

$p(2)^2 - 3q(2) - 18 = 0$

① $4p - 6q - 18 = 0$

$2p(2) + 5q - 7 = 0$

② $4p + 5q - 7 = 0$

putting equation 1 and 2 together we have

$$4p - 6q - 18 = 0 \rightarrow 4p - 6q - 18 = 0$$
$$-1(4p + 5q - 7 = 0) \rightarrow -4p - 5q + 7 = 0$$

$$-11q - 11 = 0$$
$$-11q = 11$$
$$q = -1$$

$$4p + 5(-1) - 7 = 0$$
$$4p - 12 = 0$$
$$4p = 12$$
$$p = 3$$

8) $g(x) = \dfrac{8x^3 - 1}{2x^2 + 5x - 3}$

Find your trouble spots: $2x^2 + 5x - 3 = 0$

$$(2x-1)(x+3) = 0$$
$$2x - 1 = 0 \qquad x + 3 = 0$$
$$x = \tfrac{1}{2} \qquad x = -3$$

Discontinuities @ $x = \frac{1}{2}$ and $x = -3$.

$x = \frac{1}{2}$

$$8\left(\tfrac{1}{2}\right)^3 - 1$$
$$8\left(\tfrac{1}{8}\right) - 1$$
$$1 - 1$$
$$0$$

∴ removable disc. @ $x = \frac{1}{2}$

$x = -3$

$$8(-3)^3 - 1$$
$$8(-27) - 1$$
$$\neq 0$$

∴ infinite disc. @ $x = -3$