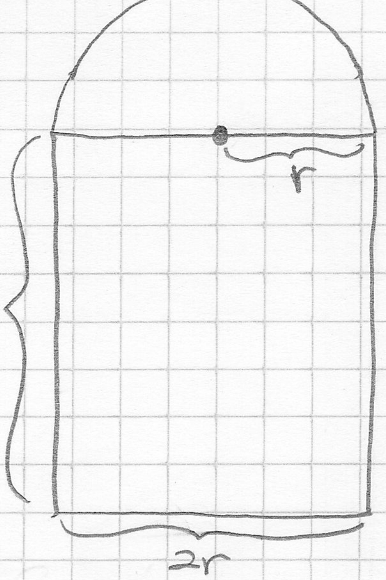
7)



Maximize Area of window

$$\underset{(A_w)}{\text{Area of window}} = \left(\begin{array}{c}\text{Area of}\\ \text{semicircle}\end{array}\right) + \left(\begin{array}{c}\text{Area of}\\ \text{rectangle}\end{array}\right)$$

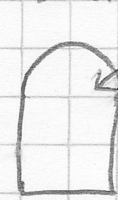
$$A_w = \underbrace{\tfrac{1}{2}\pi r^2}_{\text{semi circle}} + \underbrace{(2r)(h)}_{\text{rectangle } (l \cdot w)}$$

Problem: 2 variables, $r + h$
solve <u>another</u> formula for h.

Perimeter of window = 288 in.

$$\underset{(P_w)}{\text{Perimeter of window}} = \left(\begin{array}{c}\text{3 sides of}\\ \text{rectangle}\end{array}\right) + \left(\begin{array}{c}\text{Circumference of}\\ \text{semicircle}\end{array}\right)$$

Q: why only 3 sides?
A: b/c window has



shape

$$P_w = \underbrace{2r + h + h}_{\text{3 sides of rect.}} + \underbrace{\pi r}_{\text{circumference of semi-circle}}$$

subst. $P_w = 288$

$$288 = 2r + 2h + \pi r \Rightarrow 2h = 288 - 2r - \pi r$$

$$h = \frac{288}{2} - \frac{2r}{2} - \frac{\pi r}{2}$$

$$h = 144 - r - \tfrac{1}{2}\pi r$$

Substitute h into $A_w = \frac{1}{2}\pi r^2 + 2r\left(144 - r - \frac{1}{2}\pi r\right)$

$$A_w = \underline{\tfrac{1}{2}\pi r^2} + 288r - 2r^2 - \underline{\pi r^2}$$

$$A_w = -\tfrac{1}{2}\pi r^2 - 2r^2 + 288r$$

derivative: $\frac{dA}{dr} = -\frac{1}{2}\pi(2r) - 4r + 288 = -\pi r - 4r + 288$

critical pts:

$$-\pi r - 4r + 288 = 0$$

$$r(-\pi - 4) = -288$$

$$r = \frac{-288}{} = 40.327$$

Second deri test for local extrema

$$\frac{d^2A}{dr^2} = -\pi - 4 \quad \text{is } \underline{\text{always}} \text{ neg.}$$

$$\therefore \left.\frac{d^2A}{dr^2}\right|_{r=40.327} = (-)$$

$$\therefore \text{ max @ } r = 40.327$$

$\boxed{\text{Radius is } 40.327 \text{ in}}$

8) $R = \dfrac{V_0^2 \sin 2\theta}{g}$ V_0 and g are constants, meaning they are coefficients in our problem

$$R = \frac{V_0^2}{g} \sin 2\theta$$

$$\frac{dR}{d\theta} = \frac{V_0^2}{g} \cos(2\theta) \cdot (2) = \frac{2V_0^2}{g} \cos 2\theta$$

critical pts:

$$\frac{2V_0^2}{g} \cos 2\theta = 0$$

$$\cos 2\theta = 0$$

divide by 2

$$2\theta = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \ldots$$

$$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$$

However, $\theta = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$ makes no sense b/c angles are too big.

$\therefore$ only critical pt. worth considering is $\theta = \frac{\pi}{4}$

0	$\boxed{\frac{\pi}{4}}$	$\frac{\pi}{2}$
$+$		$-$

$\therefore$ $\boxed{\text{max @ } \theta = \frac{\pi}{4}}$

9) $P = x^3 - 48x^2 + 720x - 1000$ $[0, 40]$

Optimize Profit $\rightarrow$ Maximize P

$$\frac{dP}{dx} = 3x^2 - 96x + 720$$

critical pts:

$3x^2 - 96x + 720 = 0$

$3(x^2 - 32x + 240) = 0$

$3(x-20)(x-12) = 0$

$x - 20 = 0 \qquad x - 12 = 0$

$x = 20 \qquad x = 12$

Max. Profit. Test for Abs. Max. by plugging in critical pts <u>and</u> endpoints

$$\left.\begin{aligned} P(20) &= 2200 \\ P(12) &= 2456 \\ P(0) &= -1000 \\ P(40) &= 15{,}000 \end{aligned}\right\} \text{use a calculator}$$

$\boxed{\therefore \text{ maximum Profit is } 15{,}000 \text{ million dollars or } \$15{,}000{,}000{,}000.}$