Practice Problem Set 13

1) $x(t) = t^3 - 9t^2 + 24t$, $t > 0$

$v(t) = 3t^2 - 18t + 24$

$a(t) = 6t - 18$

3) $x(t) = \dfrac{t}{t^2+9}$, $t > 0$

direction $\longrightarrow$ velocity

$$v(t) = \frac{(1)(t^2+9) - (2t)(t)}{(t^2+9)^2} = \frac{t^2+9-2t^2}{(t^2+9)^2} = \frac{-t^2+9}{(t^2+9)^2}$$

in order for a particle to change directions, it must first come to a stop. ($v = 0$)

$$\frac{-t^2+9}{(t^2+9)^2} = 0$$

$$-t^2 + 9 = 0$$

$$t^2 = 9$$

$t = \pm 3$ but only $t = 3$ makes sense. Check if it changes directions

$v(t)$:		3	
	+		−

The particle changes directions @ $t = 3$ b/c the velocity changes signs

5) $X(t) = 3t^2 + 2t + 4, \quad t > 0$

dist. from $t=2$ to $t=5$

distance $\longrightarrow$ change in direction $\longrightarrow$ velocity

$V(t) = 6t + 2$

$6t + 2 = 0$

$6t = -2$

$t = -\frac{1}{3}$ makes <u>No</u> sense $(t > 0)$

Since velocity never $= 0$ then the particle never stops, meaning that the particle is always travelling in one direction. The direction is not important to answer a distance question.

Distance = Final position − intial position

$X(5) - X(2)$ (from $t=2$ to $t=5$)

$89 - 20$

69

The distance the particle traveled from $t=2$ to $t=5$ is 69 units.

← interesting fact, not important

Note: Since $X(5) - X(2) =$ <u>positive</u> 69 it means that the particle is moving in the positive direction.

7) $x(t) = 2\sin^2 t + 2\cos^2 t, \quad t > 0$

$x(t) = 2(\sin t)^2 + 2(\cos t)^2$

$v(t) = 4(\sin t)(\cos t) + 4(\cos t)(-\sin t)$

$v(t) = 4\sin t\cos t - 4\sin t\cos t = 0$

$a(t) = 0$

Pretty Weird!

Note: $x(t)$ can actually be written as

not important

$x(t) = 2(\sin^2 t + \cos^2 t)$

$x(t) = 2(1)$ b/c $\sin^2\theta + \cos^2\theta = 1$

$x(t) = 2$

8) $x(t) = t^3 + 8t^2 - 2t + 4, \quad t > 0$

direction $\longrightarrow$ velocity

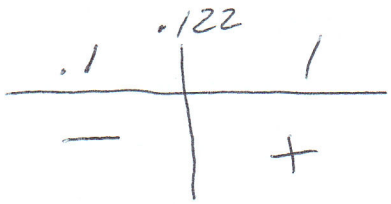
$v(t) = 3t^2 + 16t - 2$

$t = \dfrac{-16 \pm \sqrt{256 + 24}}{6} = \dfrac{-16 \pm \sqrt{280}}{6}$

$t = \dfrac{-16 + \sqrt{280}}{6}$ or $t = \dfrac{-16 - \sqrt{280}}{6}$

$t = 0.122$

makes No sense.



The particle changes direction @ $t = 0.122$ b/c $v(t)$ changes signs.

9) $x(t) = 2t^3 - 6t^2 + 12t - 18, \quad t > 0$

$v(t) = 6t^2 - 12t + 12$

$6(t^2 - 2t + 2) = 0$

$t^2 - 2t + 2 = 0$

No soln. b/c $b^2 - 4ac = 4 - 4(1)(2) =$ neg number.

Since the particle never comes to a stop, the particle never changes direction.