Power Rule WS (solns)

1) $f(x) = 9x - 6 + 2x^{-1} - x^{\frac{1}{2}}$

$f'(x) = 9 - 2x^{-2} - \frac{1}{2}x^{-\frac{1}{2}}$

$f'(x) = 9 - \frac{2}{x^2} - \frac{1}{2\sqrt{x}}$

2) $f(x) = 4x^6 - 2x^3 - x^{\frac{2}{3}}$

$f'(x) = 24x^5 - 6x^2 - \frac{2}{3}x^{-\frac{1}{3}}$

$f'(x) = 24x^5 - 6x^2 - \frac{2}{3\sqrt[3]{x}}$

3) $f(x) = \frac{x^3}{x^2} - \frac{8}{x^2} + x^{-\frac{1}{2}}$

$f(x) = x - 8x^{-2} + x^{-\frac{1}{2}}$

$f'(x) = 1 + 16x^{-3} - \frac{1}{2}x^{-\frac{3}{2}}$

$f'(x) = 1 + \frac{16}{x^3} - \frac{1}{2\sqrt{x^3}}$

4) $f(x) = 2x^3 - 4x + 10x^{\frac{1}{5}} - \frac{1}{2}x^{\frac{1}{3}}$

$f'(x) = 6x^2 - 4 + 2x^{-\frac{4}{5}} - \frac{1}{6}x^{-\frac{2}{3}}$

$f'(x) = 6x^2 - 4 + \frac{2}{\sqrt[5]{x^4}} - \frac{1}{6\sqrt[3]{x^2}}$

5) $f(x) = \frac{1}{6}x^4 - \frac{1}{8}x^2 + \frac{1}{4}x - 3x^{-1} - 4x^{-\frac{1}{2}} + 2x^{-\frac{2}{3}}$

$f'(x) = \frac{4}{6}x^3 - \frac{2}{8}x + \frac{1}{4} + 3x^{-2} + 2x^{-\frac{3}{2}} - \frac{4}{3}x^{-\frac{5}{3}}$

$f'(x) = \frac{2}{3}x^3 - \frac{1}{4}x + \frac{1}{4} + \frac{3}{x^2} + \frac{2}{\sqrt{x^3}} - \frac{4}{3\sqrt[3]{x^5}}$

6) find the function you need to take the derivative of. Don't do the limit. The function is always after the minus sign that separates $f(x+h)$ from $f(x)$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \longleftarrow *$$

$f(x) = 3x^4 - 2x^3 + x^2 - 8$ (don't distribute the minus sign)

So, $\lim_{h \to 0} \frac{3(x+h)^4 - 2(x+h)^3 + (x+h)^2 - 8 - (3x^4 - 2x^3 + x^2 - 8)}{h}$

$f'(x) = \boxed{12x^3 - 6x^2 + 2x}$

7) Again find the function

$f(x) = 2x^6 - \frac{4}{x} + \frac{2}{\sqrt{x}}$

$f(x) = 2x^6 - 4x^{-1} + 2x^{-\frac{1}{2}}$

$f'(x) = 12x^5 + 4x^{-2} - 1x^{-\frac{3}{2}}$

$f'(x) = \boxed{12x^5 + \frac{4}{x^2} - \frac{1}{\sqrt{x^3}}}$

8) $g(x) = 3x^3 - \frac{x^2}{2} + 7x - 10$

$g'(x) = 9x^2 - x + 7 \longleftarrow$ the derivative is the equation to find the slope of any tangent line @ any x-value

the slope of the tangent line @ $x=1$ $= g'(1) = 9(1)^2 - 1 + 7 = 15$

9) $g(x) = x^3 - 27x$

$g'(x) = 3x^2 - 27 = 0$ (set = to zero b/c the slope of the tangent line @ a rel. max/min = 0)

$3x^2 = 27$

$x^2 = 9$

$x = \pm 3$

sign chart

	-3		3	
-5		0		5
$3(-5)^2-27$		$3(0)^2-27$		$3(5)^2-27$
$+$		$-$		$+$

∴ g(x) has a rel. max @ $x = -3$ b/c $g'(x)$ changes from pos. to neg. $g(x)$ has a rel. min @ $x = 3$ b/c $g'(x)$ changes from neg to pos.

10) $h(x) = \frac{x^3}{3} - 3x^2 - 16x + 8$

$h'(x) = x^2 - 6x - 16$

$x^2 - 6x - 16 = 0$

$(x-8)(x+2) = 0$

$x = 8 \quad x = -2$

sign chart

	-2		8	
-3		0		10
$(-3)^2-6(-3)-16$		$(0)^2-6(0)-16$		$(10)^2-6(10)-16$
$+$		$-$		$+$

∴ $h(x)$ has a rel. max @ $x = -2$ b/c $h'(x)$ changes from + to − and $h(x)$ has a rel. min @ $x = 8$ b/c $h'(x)$ changes from − to +.