Particle Motion HW

1) $x(t) = \sin(2t) + \cos(t)$

$v(t) = x'(t) = \cos(2t)(2) - \sin(t) = 2\cos(2t) - \sin(t)$

$a(t) = v'(t) = -4\sin(2t) - \cos(t)$

2) $x(t) = \sin\left(\frac{t}{2}\right)$, $0 < t < 4\pi$

* Note: When the angle, $\frac{t}{2}$, is different than the variable in the domain, t, then I like to find the domain of the angle. (FYI: Not a necessary step but may be helpful)

$0 < t < 4\pi$) divide by 2

$0 < \frac{t}{2} < 2\pi$

Back to the problem: changing direction → velocity

$v(t) = \frac{1}{2}\cos\left(\frac{t}{2}\right) = 0$

$\cos\left(\frac{t}{2}\right) = 0$

Remember, when you solve a trig equation you solve for the given angle.

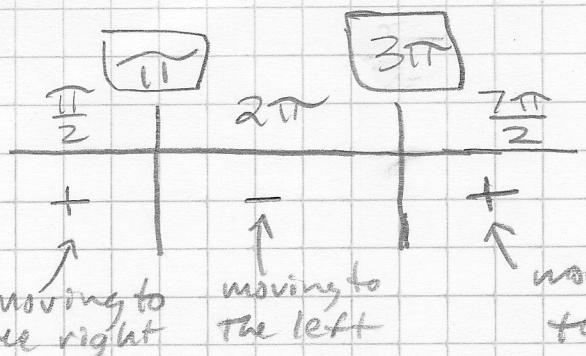
$\frac{t}{2} = \frac{\pi}{2}$ ∩ $\frac{t}{2} = \frac{3\pi}{2}$

only 2 in the domain of $0 < \frac{t}{2} < 2\pi$

$t = \pi$ ∩ $t = 3\pi$

∴ particle stops at $t = \pi$ and $t = 3\pi$

use sign chart of velocity function to find direction



∴ particle changes direction @ $t = \pi$ and $t = 3\pi$.

3) $s(t) = t^2 + 8t \qquad t > 0$

distance question $\xrightarrow{\text{uses}}$ position
velocity (direction)

$v(t) = 2t + 8 = 0$
$t = -4$ doesn't make sense b/c $t > 0$

∴ particle never comes to a stop → particle only travels in one direction

FYI: you can find out which direction by plugging any number into the velocity.

$$\frac{|}{+}$$

∴ particle is always moving to the right.

Back to the problem: the particle does not change direction from $t = 0$ to $t = 4$.

So,

$$\text{distance} = s(t_f) - s(t_i) = s(4) - s(0) = 48 - 0 = 48$$

4) $s(t) = \sin^2(2t) \qquad t > 0 \xrightarrow{\text{mult by 2}} 2t > 0$

$v(t) = 2\sin(2t)\cos(2t)(2) = 4\sin(2t)\cos(2t) = 0$

$\sin(2t) = 0$ or $\cos(2t) = 0$

infinite # of solutions to both equations, but you only care about t's between 0 and 2.

$\sin(2t) = 0$

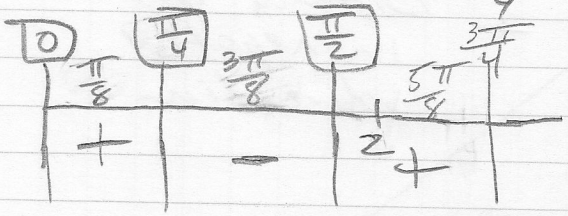
$2t = 0 \quad 2t = \pi \quad 2t = 2\pi$

$t = 0 \quad t = \frac{\pi}{2} \quad t = \pi > 2$

$\cos(2t) = 0$

$2t = \frac{\pi}{2} \quad 2t = \frac{3\pi}{2}$

$t = \frac{\pi}{4} \quad t = \frac{3\pi}{4} > 2$



Positive distance: $s\left(\frac{\pi}{4}\right) - s(0) = 1^2 - 0^2 = 1$

$s(2) - s\left(\frac{\pi}{2}\right) = \sin^2 4 - 0^2 = \sin^2 4$

Negative distance: $s\left(\frac{\pi}{2}\right) - s\left(\frac{\pi}{4}\right) = 0^2 - 1^2 = -1$

Total distance $= 1 + \sin^2 8 + |-1| = 2 + \sin^2 4$

5) $x(t) = 2t^3 - 21t^2 + 60t + 3, \quad t > 0$

speeding up $\rightarrow$ velocity and acceleration are same sign.

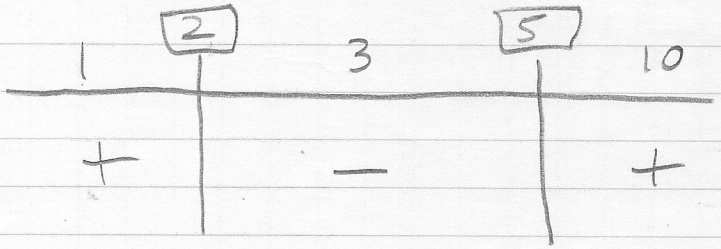
$v(t) = 6t^2 - 42t + 60 = 0$ ↙ divide by 6

$t^2 - 7t + 10 = 0$

$(t-5)(t-2) = 0$

$t - 5 = 0 \quad t - 2 = 0$

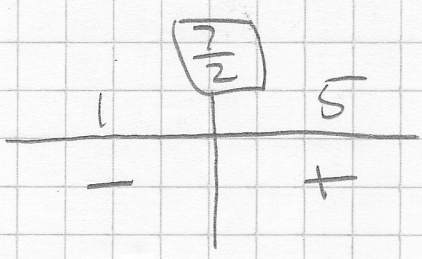
$t = 5 \quad t = 2$



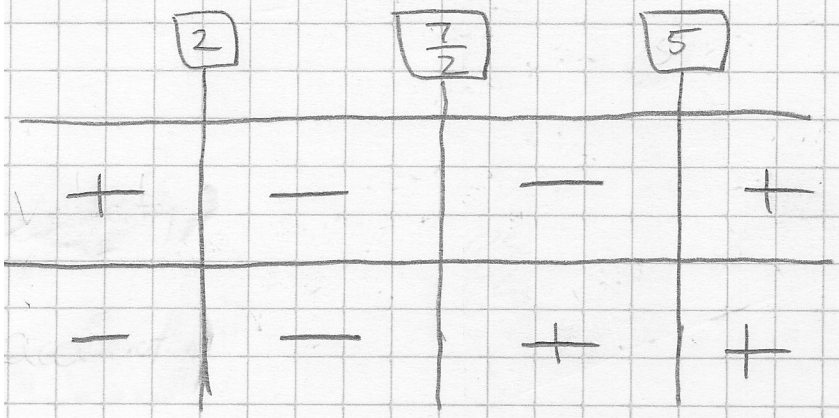
$a(t) = v'(t) = 12t - 42 = 0$

$12t = 42$

$t = \frac{7}{2}$



Trick for speeding up or slowing down questions:
Combine the tables.



∴ particle is speeding up from $t=2$ to $t=\frac{7}{2}$ and from $t=5$ on.

6) $y = \frac{x^4}{2} - 54x$

$\frac{dy}{dx} = \frac{4x^3}{2} - 54 = 2x^3 - 54 = 0$

{ x-axis is a horiz. line ($m=0$) b/c parallel lines have the same slope

$2x^3 = 54$

$x^3 = 27$

$x = 3$

∴ point is $\left(3, \frac{243}{2}\right)$

7) $A = s^2 \frac{\sqrt{3}}{4}$

Area and side length change with time, so $\frac{dA}{dt} = \frac{\sqrt{3}}{4}(2s)\frac{ds}{dt}$

looking for $\frac{dA}{dt}$.

$\frac{dA}{dt} = \frac{\sqrt{3}}{4}(2)\left(\frac{9}{1/8}\right)(27) = 243\sqrt{3}\ in^2/sec$