Optimization (Applied Max/Min) Examples

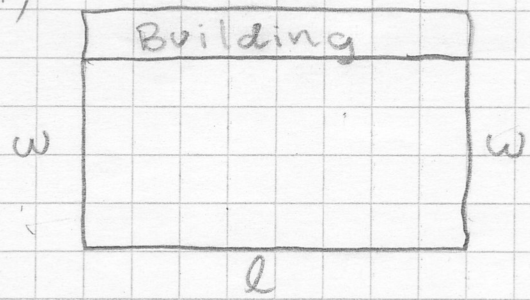
1. A rectangular field, bounded on one side by a building, is to be fenced in on the other three sides. If 3,000 feet of fence is to be used, find the dimensions of the largest field that can be fenced in. 750 ft × 1500 ft

2. A poster is to contain 100 square inches of picture surrounded by a 4-inch margin at the top and bottom and a 2-inch margin on each side. Find the overall dimensions that will minimize the total area of the poster. 22.14 in × 11.07 in

3. An open-top box with a square bottom and rectangular sides is to have a volume of 256 cubic inches. Find the dimensions that require the minimum amount of material.
8 in × 8 in × 4 in

4. Find the point on the curve $y = \sqrt{x}$ that is a minimum distance from the point (4, 0).
$\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$

1)



Largest field $\rightarrow$ Maximize Area

Rectangle:
$A = l \cdot w$

Amount of fence used $= 2w + l$

$3000 = 2w + l$

$l = 3000 - 2w$

$A = (3000 - 2w)w = 3000w - 2w^2$

$\frac{dA}{dw} = 3000 - 4w$

critical pts

$3000 - 4w = 0$

$4w = 3000$

$w = 750$

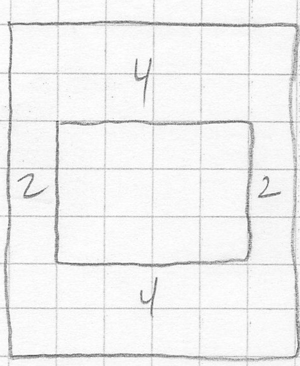
Second der. test for local extrema:

$\frac{d^2A}{dw^2} = -4$ so $\left.\frac{d^2A}{dw^2}\right|_{w=750} = -4 \Rightarrow$ local max @ $w = 750$

$l = 3000 - 2w = 3000 - 2(750) = 1500$

$\boxed{750 \text{ ft} \times 1500 \text{ ft}}$

2)



Minize Area of poster

$A = (l+4)(w+8)$

$100 = l \cdot w$

$l = \frac{100}{w}$

$$A = \left(\frac{100}{w} + 4\right)(w+8)$$

$$A = 100 + 4w + \frac{800}{w} + 32 = 132 + 4w + 800w^{-1}$$

$$\frac{dA}{dw} = 4 - \frac{800}{w^2}$$

critical pts

$$4 - \frac{800}{w^2} = 0$$

$$4 = \frac{800}{w^2}$$

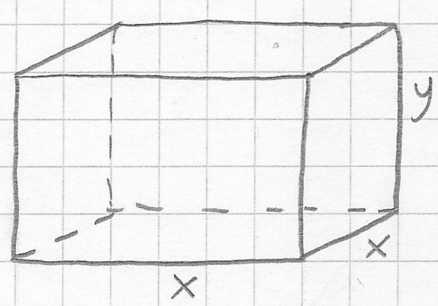
$$4w^2 = 800$$

$$w^2 = 200$$

$$w = 14.14 \qquad l = \frac{100}{w} = \frac{100}{14.14} = 7.07$$

Dimensions of poster 22.14×11.07

3.



Minimum Amt. of material
$\longrightarrow$ minimize surface area

Surface area of an open-top box
$= x^2 + xy + xy + xy + xy = x^2 + 4xy$

Volume $= 256$ in^3

$V = l \cdot w \cdot h$

$256 = x \cdot x \cdot y$

$256 = x^2 y$

$y = \frac{256}{x^2}$

$$A = x^2 + 4x\left(\frac{256}{x^2}\right) = x^2 + \frac{1024}{x}$$

$$\frac{dA}{dx} = 2x - \frac{1024}{x^2}$$

critical points

$$2x - \frac{1024}{x^2} = 0$$

$$2x = \frac{1024}{x^2}$$

$$2x^3 = 1024$$

$$x^3 = 512$$

$$x = 8$$

Second deriv. test

$$\frac{d^2A}{dx^2} = 2 + \frac{2048}{x^3}$$

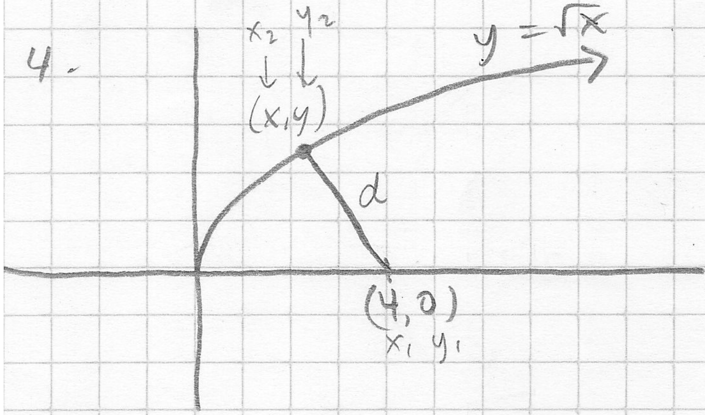
$$\left.\frac{d^2A}{dx^2}\right|_{x=8} = +$$

So, local min @ $x = 8$

$$y = \frac{256}{64} = 4$$

Dimensions are $8\text{ in} \times 8\text{ in} \times 4\text{ in}$

4.



minimum distance $\longrightarrow$ minimize distance form.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(x-4)^2 + (y-0)^2}$$

$$d = \sqrt{x^2 - 8x + 16 + y^2}$$

substitute $y = \sqrt{x}$

$$d = \sqrt{x^2 - 8x + 16 + (\sqrt{x})^2}$$

$$(d)^2 = \left(\sqrt{x^2 - 7x + 16}\right)^2$$

$$d^2 = x^2 - 7x + 16$$

$$2d\frac{dd}{dx} = 2x - 7$$

$$\frac{dd}{dx} = \frac{2x-7}{2}\left(\frac{1}{d}\right)$$

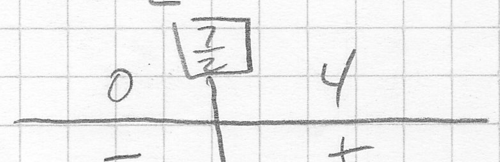
critical pts.

$$\frac{2x-7}{2} = 0 \qquad \frac{1}{d} = 0$$

No solution

$$2x - 7 = 0$$

$$x = \frac{7}{2}$$



$\therefore$ min @ $x = \frac{7}{2}$

$$y = \sqrt{x} = \sqrt{\frac{7}{2}}$$

closest point is $\left(\frac{7}{2}, \sqrt{\frac{7}{2}}\right)$