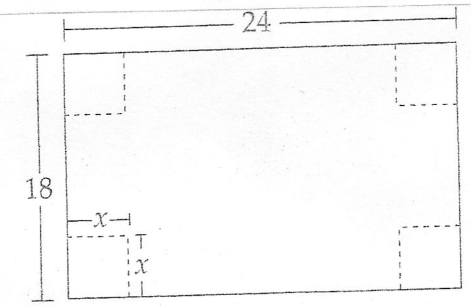
Applied Maximum/Minimum (Optimization)

1) A manufacturing company has determined that the total cost of producing an item can be determined from the equation $C = 8x^2 - 176x + 1800$, where x is the number of units that the company makes. How many units should the company manufacture in order to minimize the cost?

2) A rocket is fired into the air, and its height in meters at any given time t can be calculated using the formula $h(t) = 1600 + 196t - 4.9t^2$. Find the maximum height of the rocket and the time at which it occurs.

3) Max wants to make a box with no lid from a rectangular sheet of cardboard that is 18 inches by 24 inches. The box is to be made by cutting a square of side x from each corner of the sheet and folding up the sides (see figure below). Find the value of x that maximizes the volume of the box.



4) Find the absolute maximum and minimum values of $y = x^3 - x$ On the interval [-3,3].

5) A rectangle is to be inscribed in a semicircle with the radius 4, with one side on the semicircle's diameter. What is the largest area this rectangle can have?

Optimization

1) $C = 8x^2 - 176x + 1800$

$\frac{dc}{dx} = 16x - 176$

critical pts

$\frac{dc}{dx} = 0: \quad 16x - 176 = 0$

$16x = 176$

$x = 11$

$\frac{dc}{dx} = \text{undef}:$ No soln

max or min? — * Show 2nd der. test for max/min

$\boxed{11}$

0	12
$-$	$+$

∴ local min @ 11

∴ company should manufacture 11 units in order to minimize cost

2) $h(t) = 1600 + 196t - 4.9t^2$

$h'(t) = 196 - 9.8t$

critical pts

$196 - 9.8t = 0$

$9.8t = 196$

$t = 20$

2nd der test: $h''(t) = -9.8$

$h''(20) = -9.8$

∴ local max @ $t = 20s$

Asked for max height.

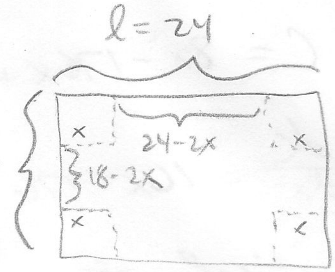
Plug 20 into original to find max value

3) Popular Question:

Come up w/ equation on your own

Volume of a box $\quad V = \underbrace{l \cdot w \cdot h}$

can't write eqn in 3 variables has to be 1.



$$V = \overset{l}{(24-2x)}\overset{w}{(18-2x)}\overset{h}{(x)}$$

$$V = (432 - 84x + 4x^2)x = 432x - 84x^2 + 4x^3$$

$$\frac{dV}{dx} = 432 - 168x + 12x^2 = 0$$

$$12(x^2 - 14x + 36) = 0$$

$$x = \frac{14 \pm \sqrt{196-144}}{2} = \frac{14 \pm \sqrt{52}}{2} = \frac{14 \pm 2\sqrt{13}}{2} = 7 \pm \sqrt{13} \approx 3.4,\ 10.6$$

* when you get more than one answer to an application problem, it's a good idea to check them for validity

x can't be 10.6 b/c it's too big for the width

Second der. test: $\dfrac{d^2V}{dx^2} = -168 + 24x$

$$\left.\frac{d^2V}{dx^2}\right|_{x=3.4} = -168 + 24(3.4) = (-)$$

$\therefore$ local max @ $x = 3.4$

if they ask for the dimensions plug x in

$$l = 24 - 2x = 17.2 \text{ in}$$
$$w = 18 - 2x = 11.2 \text{ in}$$
$$h = x = 3.4 \text{ in}$$

Given: info about circle.

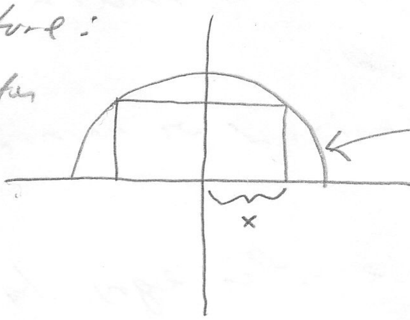
5) Eqn. of a circle: $x^2+y^2=r^2$

$x^2+y^2=16$ — full circle

✱ Using more than one eqn.

easiest picture:

always draw diameter on the x-axis unless told otherwise



Q: Know eqn. for upper half of a semi-circle

A: Do know

$y=\sqrt{16-x^2}$

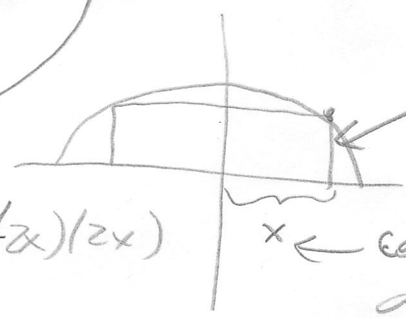
get it by solving $x^2+y^2=16$ for y.

✱ have to always write eqns in terms of a single variable

Want to max. area of rectangle:

Area of retangle = l·w

$$A(x)=\underset{l}{(2x)}\,\underset{w}{\left(\sqrt{16-x^2}\right)}$$



how do we find width, ie y.

x ← call this distance x

$l=2x$

$w=\sqrt{16-x^2}$

$$\frac{dA}{dx}=2\left(\sqrt{16-x^2}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{16-x^2}}\right)(-2x)(2x)$$

$$\frac{dA}{dx}=\frac{2(16-x^2)}{\sqrt{16-x^2}}-\frac{2x^2}{\sqrt{16-x^2}}$$

$$\frac{dA}{dx}=\frac{32-4x^2}{\sqrt{16-x^2}}$$

$\frac{dA}{dx}=0$

$32-4x^2=0$

$4x^2=32$

$x^2=8$

$x=\pm\sqrt{8}$ → in the domain of $A(x)$

$\frac{dA}{dx}=$ undef.

$\sqrt{16-x^2}=0$

$16-x^2=0$

$x^2=16$

$x=\pm4$

don't pick

-4		$-\sqrt{8}$		$+\sqrt{8}$		4
	-3		0		3	
	$-$		$+$		$-$	

don't pick not in domain of $A(x)$

∴ local max @ $x=+\sqrt{8}$

max area $=2(\sqrt{8})\left(\sqrt{16-(\sqrt{8})^2}\right)=16$