Logarithmic Differentiation WS

1) $y = x^{(e^x)}$

$\ln y = \ln x^{(e^x)}$

$\ln y = e^x \cdot \ln(x)$

$\frac{1}{y}\frac{dy}{dx} = (e^x)(\ln(x)) + (e^x)(\frac{1}{x})$

$\frac{dy}{dx} = \left(e^x \ln(x) + \frac{e^x}{x}\right)y$

$\frac{dy}{dx} = \left(e^x \ln(x) + \frac{e^x}{x}\right)\left(x^{(e^x)}\right)$

2) $y = 2^{\sin^{-1}(x)}$

$\frac{dy}{dx} = 2^{\sin^{-1}(x)} \cdot \ln 2 \left(\frac{1}{\sqrt{1-x^2}}\right) = \frac{2^{\sin^{-1}(x)} \cdot \ln 2}{\sqrt{1-x^2}}$

3) $y = (1+x)^{\frac{1}{x}}$

$\ln y = \ln(1+x)^{\frac{1}{x}}$

$\ln y = \left(\frac{1}{x}\right) \cdot \ln(1+x)$

$\frac{1}{y}\frac{dy}{dx} = (-x^{-2})(\ln(1+x)) + \left(\frac{1}{x}\right)\left(\frac{1}{1+x}\right)(1)$

$\frac{dy}{dx} = \left(-\frac{\ln(1+x)}{x^2} + \frac{1}{x(1+x)}\right)y$

$\frac{dy}{dx} = \left(-\frac{\ln(1+x)}{x^2} + \frac{1}{x(1+x)}\right)(1+x)^{\frac{1}{x}}$

4) $y = e^{\ln(x^2+1)}$

$$\frac{dy}{dx} = e^{\ln(x^2+1)}\left(\frac{1}{x^2+1}\right)(2x)$$

$$\frac{dy}{dx} = \frac{2x\,e^{\ln(x^2+1)}}{x^2+1}$$

5) $y = (\ln x)^{\ln x}$

$$\ln y = \ln(\ln x)^{\ln x}$$

$$\ln y = \ln x \cdot \ln(\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = \left(\frac{1}{x}\right)(\ln(\ln x)) + \left(\frac{1}{\ln x}\right)\left(\frac{1}{x}\right)(\ln x)$$

$$\frac{dy}{dx} = \left(\frac{\ln(\ln x)}{x} + \frac{1}{x}\right)y$$

$$\frac{dy}{dx} = \left(\frac{\ln(\ln x) + 1}{x}\right)\left((\ln x)^{\ln x}\right)$$

6) $y = \left(\frac{1+x}{1-x}\right)^x$

$$\ln y = \ln\left(\frac{1+x}{1-x}\right)^x$$

$$\ln y = x \cdot \ln\left(\frac{1+x}{1-x}\right)$$

$1-x+1+x$

$$\frac{1}{y}\frac{dy}{dx} = (1)\cdot\left(\ln\left(\frac{1+x}{1-x}\right)\right) + \left(\frac{1}{\frac{1+x}{1-x}}\right)\left(\frac{(1)(1-x)-(-1)(1+x)}{(1-x)^2}\right)(x)$$

$$\frac{dy}{dx} = \left[\ln\left(\frac{1+x}{1-x}\right) + \left(\frac{1-x}{1+x}\right)\left(\frac{2}{(1-x)^2}\right)(x)\right]y$$

$$\frac{dy}{dx} = \left[\ln\left(\frac{1+x}{1-x}\right) + \frac{2x}{(1+x)(1-x)}\right]\left(\frac{1+x}{1-x}\right)^x$$