Integral as a Net Change

The integral is a tool that can be used to calculate net change (difference between final and initial values), for example, displacement and it can also calculate total accumulation, for example, distance. Integrals can be used to calculate growth, decay and consumption. Whenever you want to find the cumulative effect of a varying rate of change, integrate it.

Ex 1| The rate of potato consumption is given by $C(t) = 2.2 + 1.1^t$ in millions of bushels per year.

Q: What does $\int_2^4 C(t)\,dt$ represent?

A: The net change in millions of bushels consumed from $t = 2$ to $t = 4$

Q: What does $\int_2^4 |C(t)|\,dt$ represent?

A: The total in millions of bushels

$\boxed{\text{Ex 2}}$ The rate, in gallons per minute, at which a pump brings water into a reservoir is given by $R(t)$.

Q: What does $\int_0^{60} R(t)$ represent?

A: The net change in the number of gallons of water pumped into the reservoir in the 1st hour.

Q: What does $\int_0^{60} |R(t)|\,dt$ represent?

A: The total number of gallons pumped into the reservoir in the 1st hour.

Ex. 3 $P(t)$ represents the rate at which the population is changing at time, t, in minutes.

Q: What does $\int_{t_1}^{t_2} P(t)\,dt$ mean?

A: The net change of the population in people from $t = t_1$ to $t = t_2$.

Q: What does $\int_{t_1}^{t_2} |P(t)|\,dt$ mean?

A: The total population, in people from $t = t_1$ to $t = t_2$.

Q: What does $\dfrac{\int_{t_1}^{t_2} P(t)\,dt}{t_2 - t_1}$ represent?

A: The average population in people/minute from $t = t_1$ to $t = t_2$

Extra Question for Example 1: What does $\frac{1}{2}\int_2^4 C(t)\,dt$ mean?

A: The average value of potato consumption in millions of bushels per year from $t = 2$ to $t = 4$.

Extra Question for Example 2: What does $\frac{1}{60}\int_0^{60} R(t)$ represent?

A: The average value of water being pumped into the reservoir in gallons/min from $t = 0$ to $t = 60$.