

Inc, Dec, Concavity WS 2 (solns)

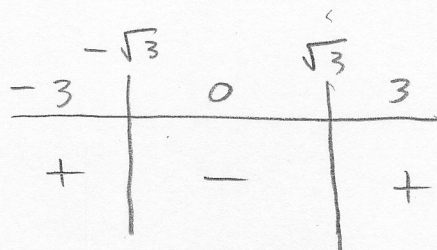
1) $y = x^3 - 9x - 6$

$$\frac{dy}{dx} = 3x^2 - 9 = 0$$

$$3x^2 = 9$$

$$x^2 = 3$$

$$x = \pm\sqrt{3}$$



$\therefore y$ is inc on $(-\infty, -\sqrt{3}]$, $[\sqrt{3}, +\infty)$
b/c $\frac{dy}{dx}$ is +.

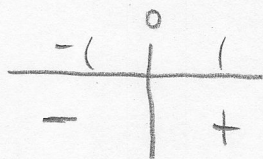
y is dec on $[-\sqrt{3}, \sqrt{3}]$ b/c $\frac{dy}{dx}$ is -

y has a rel. max @ $x = -\sqrt{3}$ b/c $\frac{dy}{dx}$ changes from + to -

y has a rel. min. @ $x = \sqrt{3}$ b/c $\frac{dy}{dx}$ changes from - to +.

$$\frac{d^2y}{dx^2} = 6x = 0$$

$$x = 0$$



$\therefore y$ is concave down on $(-\infty, 0)$ b/c $\frac{d^2y}{dx^2}$ is -

y is concave up on $(0, +\infty)$ b/c $\frac{d^2y}{dx^2}$ is +.

y has an inflection pt. @ $x = 0$ b/c $\frac{d^2y}{dx^2}$ changes from - to +.

2) $y = -x^3 - 6x^2 - 9x - 4$

$$\frac{dy}{dx} = -3x^2 - 12x - 9 = 0$$

$$-3(x^2 + 4x + 3) = 0$$

$$-3(x+3)(x+1) = 0$$

$$x+3=0, x+1=0$$

$$x=-3 \quad x=-1$$

-3	-1	
-5	-2	0
-	+	-

$\therefore y$ is dec on $(-\infty, -3], [-1, +\infty)$ b/c y' is -

y is inc on $[-3, -1]$ b/c y' is +

$$y'' = -6x - 12 = 0$$

$$-6x = 12$$

$$x = -2$$

-2	
-3	0
+	-

$\therefore y$ is concave up on $(-\infty, -2)$ b/c y'' is +

y is concave down on $(-2, 0)$ b/c y'' is -

y has an inflection pt. @ $x = -2$ b/c y'' changes from + to -

3)

$$y = (x^2 - 4)(9 - x^2)$$

$$\frac{dy}{dx} = (2x)(9 - x^2) + (-2x)(x^2 - 4)$$

$$= 18x - 2x^3 - 2x^3 + 8x$$

$$y' = 26x - 4x^3 = 0$$

$$x(26 - 4x^2) = 0$$

$$x = 0$$

$$26 - 4x^2 = 0$$

$$4x^2 = 26$$

$$x^2 = \frac{26}{4}$$

$$x = \pm \frac{\sqrt{26}}{2}$$

$-\frac{\sqrt{26}}{2}$	0	$\frac{\sqrt{26}}{2}$	
-5	-1	1	5
+	-	+	-

$\therefore y$ is inc on $(-\infty, -\frac{\sqrt{26}}{2}], [0, \frac{\sqrt{26}}{2}]$

b/c y' is +.

y is dec. on $[-\frac{\sqrt{26}}{2}, 0], [\frac{\sqrt{26}}{2}, +\infty)$

b/c y' is -.

y has a rel. max @ $x = -\frac{\sqrt{26}}{2}, \frac{\sqrt{26}}{2}$ b/c $\frac{dy}{dx}$ changes from + to -.

y has a rel. min @ $x = 0$ b/c $\frac{dy}{dx}$ changes from - to +.

$$y'' = 26 - 12x^2 = 0$$

$$26 = 12x^2$$

$$x^2 = \frac{26}{12}$$

$$x = \pm \sqrt{\frac{26}{12}}$$

$-\sqrt{\frac{26}{12}}$		$\sqrt{\frac{26}{12}}$
-3	0	3
-	+	-

$\therefore y$ is concave down on $(-\infty, -\sqrt{\frac{26}{12}}), (\sqrt{\frac{26}{12}}, +\infty)$

b/c $\frac{dy}{dx^2}$ is -

y is concave up on $(-\sqrt{\frac{26}{12}}, \sqrt{\frac{26}{12}})$ b/c $\frac{dy}{dx^2}$ is +

y has inflection points @ $x = -\sqrt{\frac{26}{12}}$ b/c y'' changes from neg to pos and @ $x = \sqrt{\frac{26}{12}}$ b/c y'' changes from + to -.

4) $y = \frac{x^4}{4} - 2x^2$

$$\frac{dy}{dx} = \frac{4x^3}{4} - 4x = x^3 - 4x = 0$$

$$x(x^2 - 4) = 0$$

$$x = 0, x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

	-2		0		2
-3		-1		1	3
-		+		-	+

$\therefore y$ is dec on $(-\infty, -2], [0, 2]$ b/c y' is neg.

y is inc on $[-2, 0], [2, +\infty)$ b/c y' is pos.

y has rel. mins. @ $x = -2, 2$ b/c y' changes from neg. to pos.

y has a rel max. @ $x = 0$ b/c y' changes from + to -.

$$\frac{d^2y}{dx^2} = 3x^2 - 4 = 0$$

$$3x^2 = 4$$

$$x^2 = \frac{4}{3} \Rightarrow x = \pm \frac{2}{\sqrt{3}}$$

	$-\frac{2}{\sqrt{3}}$		$\frac{2}{\sqrt{3}}$
-3		0	3
+		-	+

$\therefore y$ is concave up on $(-\infty, -\frac{2}{\sqrt{3}}), (\frac{2}{\sqrt{3}}, +\infty)$ b/c y'' is +
 y is concave down on $(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$ b/c y'' is -
 y has inflection pts @ $x = -\frac{2}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$ b/c y''

changes signs

5) $y = xe^{x^2}$

$$\frac{dy}{dx} = (1)(e^{x^2}) + (x)(e^{x^2})(2x) = e^{x^2} + 2x^2e^{x^2} = 0$$

1st der sign chart:

0 ← pick any number to test

+

$$e^{x^2}(1 + 2x^2) = 0$$

$$e^{x^2} = 0 \quad 1 + 2x^2 = 0$$

$$\ln e^{x^2} = \ln 0$$

No soln.

$$2x^2 = -1$$

$$x^2 = -\frac{1}{2}$$

No soln.

$\therefore y$ is always inc. (globally inc) b/c y' is always +.
 No relative extrema b/c y never decreases.

$$\frac{d^2y}{dx^2} = 2xe^{x^2} + (4x)(e^{x^2}) + (2x^2)(e^{x^2})(2x)$$

$$= 2xe^{x^2} + 4xe^{x^2} + 4x^3e^{x^2}$$

$$= 6xe^{x^2} + 4x^3e^{x^2} = 0$$

$$2xe^{x^2}(3 + 2x^2) = 0$$

$$2x = 0$$

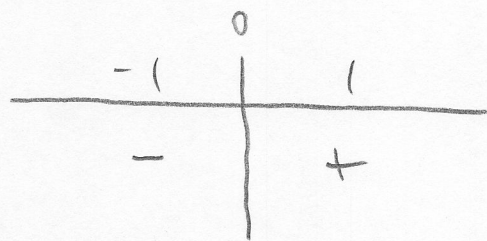
$$e^{x^2} = 0$$

$$3 + 2x^2 = 0$$

$$x = 0$$

No soln

No soln.



$\therefore y$ is concave down on $(-\infty, 0)$ b/c y'' is neg.

y is concave up on $(0, +\infty)$ b/c y'' is pos.

y has an inflection point @ $x = 0$ b/c y'' changes from - to +.