Inc, Dec, Concavity WS 2 (solns)

1) $y = x^3 - 9x - 6$

$\frac{dy}{dx} = 3x^2 - 9 = 0$

$3x^2 = 9$

$x^2 = 3$

$x = \pm\sqrt{3}$

	$-\sqrt{3}$		$\sqrt{3}$	
-3		0		3
$+$		$-$		$+$

$\therefore$ y is inc on $(-\infty, -\sqrt{3}]$, $[\sqrt{3}, +\infty)$ b/c $\frac{dy}{dx}$ is $+$.

y is dec on $[-\sqrt{3}, \sqrt{3}]$ b/c $\frac{dy}{dx}$ is $-$

y has a rel. max @ $x = -\sqrt{3}$ b/c $\frac{dy}{dx}$ changes from $+$ to $-$

y has a rel. min. @ $x = \sqrt{3}$ b/c $\frac{dy}{dx}$ changes from $-$ to $+$.

$\frac{d^2y}{dx^2} = 6x = 0$

$x = 0$

	0	
-1		1
$-$		$+$

$\therefore$ y is concave down on $(-\infty, 0)$ b/c $\frac{d^2y}{dx^2}$ is $-$

y is concave up on $(0, +\infty)$ b/c $\frac{d^2y}{dx^2}$ is $+$.

y has an inflection pt. @ $x = 0$ b/c $\frac{d^2y}{dx^2}$ changes from $-$ to $+$.

2) $y = -x^3 - 6x^2 - 9x - 4$

$\frac{dy}{dx} = -3x^2 - 12x - 9 = 0$

$-3(x^2 + 4x + 3) = 0$

$-3(x+3)(x+1) = 0$

$x+3=0$, $x+1=0$

$x=-3$ $\qquad$ $x=-1$

	-3		-1	
-5		-2		0
$-$		$+$		$-$

$\therefore$ y is dec on $(-\infty,-3]$, $[-1,+\infty)$ b/c y' is $-$

y is inc on $[-3,-1]$ b/c y' is $+$

$y''=-6x-12=0$

$-6x=12$

$x=-2$

	-2	
-3		0
$+$		$-$

$\therefore$ y is concave up on $(-\infty,-2)$ b/c y'' is $+$

y is concave down on $(-2,0)$ b/c y'' is $-$

y has an inflection pt. @ $x=-2$ b/c y'' changes from $+$ to $-$

3) $y=(x^2-4)(9-x^2)$

$\frac{dy}{dx}=(2x)(9-x^2)+(-2x)(x^2-4)$

$=18x-2x^3-2x^3+8x$

$y'=26x-4x^3=0$

$x(26-4x^2)=0$

$x=0$ $\qquad$ $26-4x^2=0$

$4x^2=26$

$x^2=\frac{26}{4}$

$x=\pm\frac{\sqrt{26}}{2}$

	$-\frac{\sqrt{26}}{2}$		0		$\frac{\sqrt{26}}{2}$	
-5		-1		1		5
$+$		$-$		$+$		$-$

$\therefore$ y is inc on $(-\infty,-\frac{\sqrt{26}}{2}]$, $[0,\frac{\sqrt{26}}{2}]$ b/c y' is $+$.

y is dec. on $[-\frac{\sqrt{26}}{2},0]$, $[\frac{\sqrt{26}}{2},+\infty)$ b/c y' is $-$.

y has a rel. max @ $x=-\frac{\sqrt{26}}{2}$, $\frac{\sqrt{26}}{2}$ b/c $\frac{dy}{dx}$ changes from $+$ to $-$.

y has a rel. min @ $x=0$ b/c $\frac{dy}{dx}$ changes from $-$ to $+$.

$$y'' = 26 - 12x^2 = 0$$

$$26 = 12x^2$$

$$x^2 = \frac{26}{12}$$

$$x = \pm\sqrt{\frac{26}{12}}$$

$-\sqrt{\frac{26}{12}}$ $\sqrt{\frac{26}{12}}$

-3	0	3
$-$	$+$	$-$

$\therefore$ y is concave down on $\left(-\infty, -\sqrt{\frac{26}{12}}\right), \left(\sqrt{\frac{26}{12}}, +\infty\right)$
b/c $\frac{d^2y}{dx^2}$ is $-$

y is concave up on $\left(-\sqrt{\frac{26}{12}}, \sqrt{\frac{26}{12}}\right)$ b/c $\frac{d^2y}{dx^2}$ is $+$

y has inflection points @ $x = -\sqrt{\frac{26}{12}}$ b/c y'' changes from neg to pos and @ $x = \sqrt{\frac{26}{12}}$ b/c y'' changes from $+$ to $-$.

4) $y = \frac{x^4}{4} - 2x^2$

$$\frac{dy}{dx} = \frac{4x^3}{4} - 4x = x^3 - 4x = 0$$

$$x(x^2-4) = 0$$

$$x = 0, \quad x^2 - 4 = 0$$

$$x^2 = 4$$

$$x = \pm 2$$

-2 0 2

-3	-1	1	3
$-$	$+$	$-$	$+$

$\therefore$ y is dec on $(-\infty, -2], [0, 2]$ b/c y' is neg.
y is inc on $[-2, 0], [2, +\infty)$ b/c y' is pos.
y has rel. mins. @ $x = -2, 2$ b/c y' changes from neg. to pos.
y has a rel max. @ $x = 0$ b/c y' changes from $+$ to $-$.

$$\frac{d^2y}{dx^2} = 3x^2 - 4 = 0$$

$$3x^2 = 4$$

$$x^2 = \frac{4}{3} \Rightarrow x = \pm\frac{2}{\sqrt{3}}$$

$-\frac{2}{\sqrt{3}}$ $\frac{2}{\sqrt{3}}$

-3	0	3
$+$	$-$	$+$

$\therefore$ y is concave up on $\left(-\infty, -\frac{2}{\sqrt{3}}\right), \left(\frac{2}{\sqrt{3}}, +\infty\right)$ b/c y'' is +

y is concave down on $\left(-\frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}\right)$ b/c y'' is −

y has inflection pts @ $x = -\frac{2}{\sqrt{3}}$ and $x = \frac{2}{\sqrt{3}}$ b/c y'' changes signs

5) $y = xe^{x^2}$

$$\frac{dy}{dx} = (1)(e^{x^2}) + (x)(e^{x^2})(2x) = e^{x^2} + 2x^2 e^{x^2} = 0$$

$$e^{x^2}(1 + 2x^2) = 0$$

$e^{x^2} = 0$	$1 + 2x^2 = 0$
$\ln e^{x^2} = \ln 0$	$2x^2 = -1$
No soln.	$x^2 = -\frac{1}{2}$
	No soln.

1st der sign chart:

0 ← pick any number to test

+

$\therefore$ y is always inc. (globally inc) b/c y' is always +.

No relative extrema b/c y never decreases.

$$\frac{d^2y}{dx^2} = 2xe^{x^2} + (4x)(e^{x^2}) + (2x^2)(e^{x^2})(2x)$$

$$= 2xe^{x^2} + 4xe^{x^2} + 4x^3 e^{x^2}$$

$$= 6xe^{x^2} + 4x^3 e^{x^2} = 0$$

$$2xe^{x^2}(3 + 2x^2) = 0$$

$2x = 0$	$e^{x^2} = 0$	$3 + 2x^2 = 0$
$x = 0$	No soln	No soln.

	0	
−1		1
−		+

$\therefore$ y is concave down on $(-\infty, 0)$ b/c y'' is neg.

y is concave up on $(0, +\infty)$ b/c y'' is pos.

y has an inflection point @ $x = 0$ b/c y'' changes from − to +.