Inc/Dec, Concavity WS

1) $f(x) = 4x^3 - 12x^2$

$f'(x) = 12x^2 - 24x = 0$

$12x(x-2) = 0$

$12x = 0 \qquad x - 2 = 0$

$x = 0 \qquad x = 2$

$\boxed{0} \qquad \boxed{2}$

$-1 \quad | \quad 1 \quad | \quad 3$

$+ \qquad - \qquad +$

a) $\therefore f(x)$ is inc $(-\infty, 0]$, $[2, +\infty)$ b/c first derivative is positive

$f(x)$ is dec on $[0, 2]$ b/c 1st der. is negative

$f''(x) = 24x - 24 = 0$

$24(x-1) = 0$

$x - 1 = 0$

$x = 1$

$\boxed{1}$

$0 \quad | \quad 2$

$- \qquad +$

b) $\therefore f(x)$ is concave up on $(1, +\infty)$ b/c $f''(x)$ is pos.

$f(x)$ is concave down on $(-\infty, 1)$ b/c $f''(x)$ is neg

c) $\therefore$ the x-coordinate of the inflection pt is $x = 1$ b/c there is a change in concavity at $x = 1$.

2) $f(x) = \frac{e^{2x}}{2} - x$

$$f'(x) = \tfrac{1}{2}e^{2x}(2) - 1 = e^{2x} - 1 = 0$$

$$e^{2x} = 1$$

$$\ln e^{2x} = \ln 1$$

$$2x \ln e = 0$$

$$2x(1) = 0$$

$$2x = 0$$

$$x = 0$$

$\boxed{0}$

$-1 \quad | \quad 1$

$- \quad | \quad +$

a) $\therefore$ $f(x)$ is inc on $[0, +\infty)$ b/c $f'(x)$ is pos.
$f(x)$ is dec on $(-\infty, 0]$ b/c $f'(x)$ is neg

$$f''(x) = e^{2x}(2) = 2e^{2x} = 0$$

$$e^{2x} = 0$$

$$\ln e^{2x} = \ln 0$$

No soln. means $f(x)$ is always concave up or always concave down

pick any #

0

$+$

b) $\therefore$ $f(x)$ is concave up on $(-\infty, +\infty)$ b/c $f''(x)$ is pos.

c) $\therefore$ No inflection points

3) $f(x) = \dfrac{x}{x^2+1}$

$$f'(x) = \frac{1(x^2+1) - (2x)(x)}{(x^2+1)^2} = \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$f'(x) = \frac{1-x^2}{(x^2+1)^2} = 0$$

$$1 - x^2 = 0$$
$$1 = x^2$$
$$x = \pm 1$$

-2	$\boxed{-1}$	0	$\boxed{1}$	2
$-$		$+$		$-$

a) $\therefore$ $f(x)$ is inc on $[-1, 1]$ b/c $f'(x)$ is pos
$f(x)$ is dec on $(-\infty, -1]$, $[1, +\infty)$ b/c $f'(x)$ is neg

$$f''(x) = \frac{-2x(x^2+1)^2 - 2(x^2+1)'(2x)(1-x^2)}{(x^2+1)^4}$$

$$f''(x) = \frac{-2x(x^2+1) - 4x(1-x^2)}{(x^2+1)^3}$$

cancel out (x^2+1)

$$= \frac{-2x^3 - 2x - 4x + 4x^3}{(x^2+1)^3} = \frac{2x^3 - 6x}{(x^2+1)^3} = 0$$

$$2x^3 - 6x = 0$$
$$2x(x^2 - 3) = 0$$
$$2x = 0 \qquad x^2 - 3 = 0$$
$$x = 0 \qquad x^2 = 3$$
$$x = \pm\sqrt{3}$$

-3	$\boxed{-\sqrt{3}}$	-1	$\boxed{0}$	1	$\boxed{\sqrt{3}}$	4
$-$		$+$		$-$		$+$

b) $\therefore$ concave up on $(-\sqrt{3}, 0)$, $(\sqrt{3}, +\infty)$ b/c $f''(x)$ is pos
concave down on $(-\infty, -\sqrt{3})$, $(0, \sqrt{3})$ b/c $f''(x)$ is neg

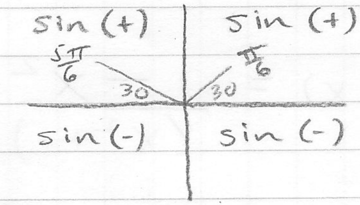
c) the x-coordinates of the inflection pts are $x = -\sqrt{3}, 0, \sqrt{3}$

4) $f(x) = 2\cos x + x$ on $[-\pi, \pi]$

$f'(x) = -2\sin x + 1 = 0$

$\sin x = \frac{1}{2}$

reference $\angle$: $\alpha = \frac{\pi}{6}$



$x = \frac{\pi}{6}, \frac{5\pi}{6}$

0	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{5\pi}{6}$	π
$+$		$-$		$+$

a) $\therefore$ $f(x)$ is inc on $[-\pi, \frac{\pi}{6}], [\frac{5\pi}{6}, \pi]$ b/c $f'(x)$ is (+)
$f(x)$ is dec on $[\frac{\pi}{6}, \frac{5\pi}{6}]$ b/c $f'(x)$ is (−)

$f''(x) = -2\cos x = 0$

$\cos x = 0$

$x = \frac{\pi}{2}, -\frac{\pi}{2}$

$-\pi$	$\boxed{-\frac{\pi}{2}}$	0	$\boxed{\frac{\pi}{2}}$	π
$+$		$-$		$+$

b) $\therefore$ $f(x)$ is concave up on $(-\pi, -\frac{\pi}{2}), (\frac{\pi}{2}, \pi)$ b/c $f''(x)$ is pos
$f(x)$ is concave down on $(-\frac{\pi}{2}, \frac{\pi}{2})$ b/c $f''(x)$ is neg

c) $\therefore$ the x-coordinates of the inflection pts are $x = -\frac{\pi}{2}, \frac{\pi}{2}$

5) $f(x) = \sin(x) - \sqrt{3}\cos(x)$ on $[0, 2\pi]$

$f'(x) = \cos(x) + \sqrt{3}\sin(x) = 0$

$$\sqrt{3}\,\frac{\sin(x)}{\cos(x)} = \frac{-\cos x}{\cos x}$$

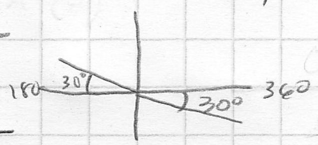
$$\sqrt{3}\,\frac{\sin(x)}{\cos(x)} = -1$$

$$\frac{\sin(x)}{\cos(x)} = -\frac{1}{\sqrt{3}} \Rightarrow$$ guess until you find the correct ref. angle

(you want $\cos(x) = \sqrt{3}$ b/c both $\cos(x)$ and $\sqrt{3}$ are in the denominator) $\Rightarrow$ if $x = 30°$ then $\frac{\sin(30°)}{\cos(30°)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$

So, reference angle $= 30° = \frac{\pi}{6}$

$\frac{\sin(x)}{\cos(x)} \to -$	$\frac{\sin(x)}{\cos(x)} \to +$
$\frac{\sin(x)}{\cos(x)} \to +$	$\frac{\sin(x)}{\cos(x)} \to -$



solutions are $150°$ and $330°$ which is $\frac{5\pi}{6}$, $\frac{11\pi}{6}$

0	$\frac{5\pi}{6}$	π	$\frac{11\pi}{6}$	2π
$+$		$-$		$+$

$\boxed{f(x) \text{ is inc on } [0, \tfrac{5\pi}{6}], [\tfrac{11\pi}{6}, 2\pi] \text{ b/c } f' \text{ is pos. } f(x) \text{ is dec on } [\tfrac{5\pi}{6}, \tfrac{11\pi}{6}] \text{ b/c } f' \text{ is neg.}}$

$f''(x) = -\sin(x) + \sqrt{3}\cos(x) = 0$

$$\frac{\sqrt{3}\cos x}{\cos x} = \frac{\sin x}{\cos x}$$

$$\sqrt{3} = \tan x$$

$$x = \frac{\pi}{3}, \frac{4\pi}{3}$$

0	$\boxed{\frac{\pi}{3}}$	π	$\boxed{\frac{4\pi}{3}}$	2π
$+$		$-$		$+$

b) $\therefore$ $f(x)$ is concave up on $(0, \frac{\pi}{3}), (\frac{4\pi}{3}, 2\pi)$ b/c $f''(x)$ is pos
$f(x)$ is concave down on $(\frac{\pi}{3}, \frac{4\pi}{3})$ b/c $f''(x)$ is neg.

c) x-coordinates for pts. of inflection: $x = \frac{\pi}{3}, \frac{4\pi}{3}$