Difference Quotient

Find the slope of the local line at the indicated point or value.

1) $f(x) = x^2 + 2$ @ $(1,3)$

2) $g(x) = 3x^2 - 2x$ @ $x = 2$

3) $h(x) = x^3 - 2x + 1$ @ $(1,0)$

Use a calculator

Then show them with the difference quotient.

$$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$$

1) $x_0 = 1$

$$\lim_{x \to 1} \frac{f(x) - f(1)}{x - 1} = \lim_{x \to 1} \frac{x^2 + 2 - 3}{x - 1} = \lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

$$= \lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = \lim_{x \to 1} x + 1 = 1 + 1 = 2$$

2) $x_0 = 2$

$$\lim_{x \to 2} \frac{g(x) - g(2)}{x - 2} = \lim_{x \to 2} \frac{3x^2 - 2x - 8}{x - 2} = \lim_{x \to 2} \frac{(3x+4)\cancel{(x-2)}}{\cancel{x-2}}$$

$$= \lim_{x \to 2} 3x + 4 = 6 + 4 = 10$$

3) $x_0 = 1$

$$\lim_{x\to 1} \frac{h(x) - h(x_0)}{x-1} = \lim_{x\to 1} \frac{x^3 - 2x + 1 - 0}{x-1} = \lim_{x\to 1} \frac{x^3 - 2x + 1}{x-1}$$

$$
\begin{array}{r}
x^2 + x - 1 \\
x-1 \overline{)\, x^3 - 2x + 1} \\
-x^3 + x^2 \\
\hline
x^2 - 2x \\
-x^2 + x \\
\hline
-x + 1 \\
+x - 1 \\
\hline
0
\end{array}
$$

$$= \lim_{x\to 1} x^2 + x - 1 = 1 + 1 - 1 = \boxed{1}$$

HW:

1) $x^2 - 3x + 4$ @ $x = -1$

2) $x^3 - 8$ @ $x = 2$

3) $x^2 + 5$ @ $x = 1$

4) $\sin x$ @ $x = \frac{\pi}{2}$

Difference Quotient day 2

Ex. 1 Find the slope of the local line for $f(x) = \frac{2}{x}$ @ $(2,1)$

$$\lim_{x \to 2} \frac{f(x) - f(2)}{x-2} = \lim_{x \to 2} \frac{\frac{2}{x} - 1}{x-2} = \frac{\frac{2-x}{x}}{x-2} = \frac{2-x}{x(x-2)} = \frac{-1}{x} = -\frac{1}{2}$$

Ex. 2 Find the equation for the slope of the local line for $f(x) = x^2 + 4$

the other "difference quotient"

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 + 4 - (x^2+4)}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 \cancel{+4} - \cancel{x^2} \cancel{-4}}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2x+h)}{h} = \lim_{h \to 0} 2x + h = 2x$$

Use the equation of the slope to find the slopes @ many different values

a) @ $x = 1$ $\quad m_{tan} = 2(1) = 2$

b) @ $x = -3$ $\quad m_{tan} = 2(-3) = -6$

c) @ $x = 4$ $\quad m_{tan} = 2(4) = 8$

d) @ $x = a$ $\quad m_{tan} = 2(a) = 2a$