Differential Eqns. WS 1

1) $\frac{dy}{dx} = \frac{7x^2}{y^3}$, $y(3) = 2$

Separate variables: $y^3\,dy = 7x^2\,dx$

$$\int y^3\,dy = \int 7x^2\,dx$$

$$\frac{y^4}{4} = \frac{7x^3}{3} + C$$

I'm choosing to leave eqn as y^4 $\longrightarrow$ $y^4 = \frac{28x^3}{3} + 4C$

Solving for C: $y(3) = 2$ (x ↓ 3, y ↓ 2)

$$2^4 = \frac{28(3)^3}{3} + 4C$$

$$16 = 252 + 4C$$

$$4C = -236$$

$$C = -59$$

Plug C back into eqn. $y^4 = \frac{28x^3}{3} + 4(-59)$

$$\boxed{y^4 = \frac{28x^3}{3} - 236}$$

2) $\frac{dy}{dx} = 5x^2y$, $y(0) = 6$

$$\frac{dy}{y} = 5x^2\,dx$$

$$\int \frac{1}{y}\,dy = \int 5x^2\,dx$$

$$\ln y = \frac{5x^3}{3} + c$$

$y(0) = 6$

$$\ln 6 = 0 + c \Rightarrow c = \ln 6$$

$$\therefore \ln y = \frac{5x^3}{3} + \ln 6$$

$$e^{\ln y} = e^{\frac{5x^3}{3} + \ln 6}$$

$$y = e^{\frac{5x^3}{3}} \cdot e^{\ln 6} = e^{\frac{5x^3}{3}} \cdot 6$$

$$\boxed{y = 6e^{\frac{5x^3}{3}}}$$

3) $\frac{dy}{dx} = \frac{1}{y + x^2 y}$, $y(0) = 2$

$$\frac{dy}{dx} = \frac{1}{y(1+x^2)}$$

$$y\,dy = \frac{1}{1+x^2}\,dx$$

$$\int y\,dy = \int \frac{1}{1+x^2}\,dx$$

$$\frac{y^2}{2} = \tan^{-1}x + c$$

$$y^2 = 2\tan^{-1}x + 2c$$

$y(0) = 2$

$$2^2 = 2\tan^{-1}(0) + 2c$$

$$4 = 2(0) + 2c \Rightarrow 4 = 2c \Rightarrow c = 2$$

$$\boxed{y^2 = 2\tan^{-1}x + 4}$$

4) $\frac{dy}{dx} = \frac{e^x}{y^2}$, $y(0) = 1$

$$y^2\,dy = e^x\,dx$$

$$\int y^2\,dy = \int e^x\,dx$$

$$\frac{y^3}{3} = e^x + C$$

$$y^3 = 3e^x + 3C$$

$y(0) = 1$

$$1^3 = 3e^0 + 3C$$

$$1 = 3 + 3C$$

$$3C = -2$$

$$C = -\frac{2}{3}$$

$$y^3 = 3e^x + 3\left(-\frac{2}{3}\right)$$

$$\boxed{y^3 = 3e^x - 2}$$

5) $\frac{dy}{dx} = \frac{y^2}{x^3}$, $y(1) = 2$

$$\frac{1}{y^2}\,dy = \frac{1}{x^3}\,dx$$

$$\int \frac{1}{y^2}\,dy = \int \frac{1}{x^3}\,dx$$

$$\frac{y^{-1}}{-1} = \frac{x^{-2}}{-2} + C$$

$$-\frac{1}{y} = -\frac{1}{2x^2} + C$$

$y(1) = 2$

$$-\frac{1}{2} = -\frac{1}{2(1)^2} + C$$

$$-\frac{1}{2} = -\frac{1}{2} + C$$

$$C = 0$$

$$\therefore -\frac{1}{y} = -\frac{1}{2x^2} + 0 = -\frac{1}{2x^2}$$

$$\frac{1}{y} = \frac{1}{2x^2}$$

$$\boxed{y = 2x^2}$$

6) $\frac{dy}{dx} = \frac{\sin x}{\cos y}$, $y(0) = \frac{3\pi}{2}$

$$\cos y \, dy = \sin x \, dx$$

$$\int \cos y \, dy = \int \sin x \, dx$$

$$\sin y = -\cos x + C$$

$$\sin\left(\frac{3\pi}{2}\right) = -\cos(0) + C$$

$$-1 = -1 + C$$

$$C = 0$$

$$\boxed{\sin y = -\cos x}$$

7) Distance $= 11 + 14 = \boxed{25}$

Displacement $= 11 - 14 = \boxed{-3}$

$$s(9) - s(0) = \int_0^9 s'(t)\,dt = \int_0^9 v(t)\,dt$$

$$s(9) = \int_0^9 v(t)\,dt + s(0) = -3 + 6 = \boxed{3}$$