

Finding derivatives of inverse functions

1) If $f(x) = x^5 + 3x - 8$, find $(f^{-1})'(-8)$

\swarrow x-value for $f(x)$
 \searrow means y-value for $f(x)$

$$y = x^5 + 3x - 8$$

$$x = y^5 + 3y - 8$$

$$-8 = y^5 + 3y - 8$$

$$0 = y^5 + 3y$$

$$0 = y(y^4 + 3)$$

$$y = 0 \quad y^4 + 3 = 0$$

No soln

$$1 = 5y \frac{dy}{dx} + 3 \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{5y + 3}$$

$$\frac{dy}{dx} = \frac{1}{5(0) + 3} = \left(\frac{1}{3}\right)$$

2) If $f(x) = \frac{8}{x^3}$, find $(f^{-1})'(1)$

$$y = \frac{8}{x^3}$$

$$x = \frac{8}{y^3} = 8y^{-3}$$

$$1 = \frac{8}{y^3} \Rightarrow y^3 = 8 \Rightarrow y = 2$$

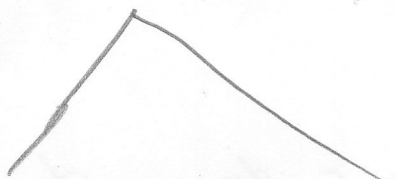
$$1 = -24y^{-4} \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{-24y^{-4}} = \frac{-y^4}{24}$$

$$\frac{dy}{dx} = \frac{-2^4}{24} = \frac{-16}{24} = \left(-\frac{2}{3}\right)$$

3) Find $(f^{-1})'(1)$ for $y = 2x - x^3$

$$x = 2y - y^3$$



$$1 = 2y - y^3$$

$$1 = 2 \frac{dy}{dx} - 3y^2 \frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{1}{2 - 3y^2} = \frac{1}{2 - 3} = -1$$

2 ways
to solve

use calculator to graph

$$y = 2x - x^3 - 1$$

and find zeros

"plug & chug"
or
guess

✓
 $y = 1$