Finding derivatives of inverse functions

1) If $f(x) = x^5 + 3x - 8$, find $(f^{-1})'_{(-8)}$. (x-value for $f^{-1}(x)$ means y-value for $f(x)$)

$y = x^5 + 3x - 8$

$x = y^5 + 3y - 8$

$-8 = y^5 + 3y - 8$

$0 = y^5 + 3y$

$0 = y(y^4 + 3)$

$y = 0 \quad y^4 + 3 = 0$

No soln

$1 = 5y\frac{dy}{dx} + 3\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{5y+3}$

$\frac{dy}{dx} = \frac{1}{5(0)+3} = \frac{1}{3}$

2) If $f(x) = \frac{8}{x^3}$, find $(f^{-1})'_{(1)}$

$y = \frac{8}{x^3}$

$x = \frac{8}{y^3} = 8y^{-3}$

$1 = \frac{8}{y^3} \Rightarrow y^3 = 8 \Rightarrow y = 2$

$1 = -24y^{-4}\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{-24y^{-4}} = \frac{-y^4}{24}$

$\frac{dy}{dx} = \frac{-2^4}{24} = \frac{-16}{24} = -\frac{2}{3}$

3) Find $(f^{-1})'(1)$ for $y = 2x - x^3$

$x = 2y - y^3$

$1 = 2y - y^3$

2 ways to solve

"Plug & chug" ie guess

$y = 1$

use calculator to graph $y = 2x - x^3 - 1$ and find zeros

$1 = 2\frac{dy}{dx} - 3y^2\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{2 - 3y^2} = \frac{1}{2-3} = -1$