Derivative of f^{-1} WS

1) $y = x + \frac{1}{x}$

$x = y + \frac{1}{y}$

$1 = \frac{dy}{dx} + -y^{-2}\frac{dy}{dx}$

$1 = \frac{dy}{dx}\left(1 - \frac{1}{y^2}\right)$

$\frac{dy}{dx} = \frac{1}{1 - \frac{1}{y^2}}$

$\frac{17}{4} = y + \frac{1}{y}$

$\frac{17}{4} = \frac{y^2+1}{y}$

$\therefore y = 4$

So, $\left.\frac{dy}{dx}\right|_{y=4} = \frac{1}{1-\frac{1}{16}} = \frac{1}{\frac{15}{16}} = \boxed{\frac{16}{15}}$

2) $y = 3x - 5x^3$ @ $y = 2$

$x = 3y - 5y^3$

$1 = 3\frac{dy}{dx} - 15y^2\frac{dy}{dx}$

$2 = 3y - 5y^3$

$\therefore y = -1$

So, $\left.\frac{dy}{dx}\right|_{y=-1} = \frac{1}{3-15} = -\frac{1}{12}$

3) $y = e^x$

$x = e^y$

method 1: We can actually solve for y in this equation

$x = e^y$

$\ln x = \ln e^y$

$\ln x = y$

$y = \ln x$ is the inverse function

$$\frac{dy}{dx} = \frac{1}{x}$$

$$\left.\frac{dy}{dx}\right|_{x=e} = \boxed{\frac{1}{e}}$$

method 2: $x = e^y$

$$1 = e^y \frac{dy}{dx} \qquad e = e^y$$

$$\therefore y = 1$$

$$\frac{dy}{dx} = \frac{1}{e^y} \qquad \text{So, } \left.\frac{dy}{dx}\right|_{y=1} = \frac{1}{e^1} = \boxed{\frac{1}{e}}$$

4) $f(x) = x^7 - 2x^5 + 2x^3$ @ $f(x) = 1$

$y = x^7 - 2x^5 + 2x^3$ $\quad y = 1$

$x = y^7 - 2y^5 + 2y^3 \longrightarrow 1 = y^7 - 2y^5 + 2y^3$

$$\therefore y = 1$$

$$1 = 7y^6 \frac{dy}{dx} - 10y^4 \frac{dy}{dx} + 6y^2 \frac{dy}{dx} \qquad \text{So, } \left.\frac{dy}{dx}\right|_{y=1} = \frac{1}{7-10+6} = \boxed{\frac{1}{3}}$$

$$\frac{dy}{dx} = \frac{1}{7y^6 - 10y + 6y^2}$$

5) $y = x + x^3$

$x = y + y^3 \longrightarrow -2 = y + y^3$

$$1 = \frac{dy}{dx} + 3y^2 \frac{dy}{dx} \qquad \therefore y = -1$$

$$\frac{dy}{dx} = \frac{1}{1+3y^2}$$

So, $\left.\frac{dy}{dx}\right|_{y=-1} = \frac{1}{1+3(-1)^2} = \boxed{\frac{1}{4}}$

6) $y = 4x - x^3$

$x = 4y - y^3 \longrightarrow 3 = 4y - y^3$

$\therefore y = 1$

$$1 = 4\frac{dy}{dx} - 3y^2\frac{dy}{dx}$$

So, $\left.\frac{dy}{dx}\right|_{y=1} = \frac{1}{4-3} = \boxed{1}$

$$\frac{dy}{dx} = \frac{1}{4-3y^2}$$

7) $y = \ln(x)$

$x = \ln(y)$

We can actually solve this equation for y.

method 1: $x = \ln(y)$

$e^x = e^{\ln(y)}$

$y = e^x \longleftarrow$ inverse function

$\frac{dy}{dx} = e^x$, $\left.\frac{dy}{dx}\right|_{x=0} = e^0 = \boxed{1}$

method 2: $x = \ln(y)$

$1 = \frac{1}{y}\frac{dy}{dx}$

$0 = \ln y$

$\therefore y = 1$

$\frac{dy}{dx} = y$

$\left.\frac{dy}{dx}\right|_{y=1} = \boxed{1}$

8) $y = \sqrt[3]{x} + \sqrt[5]{x}$

$x = \sqrt[3]{y} + \sqrt[5]{y}$

$1 = \frac{1}{3} y^{-2/3} \frac{dy}{dx} + \frac{1}{5} y^{-4/5} \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{\left(\frac{1}{3} y^{-2/3} + \frac{1}{5} y^{-4/5}\right)}$

$2 = \sqrt[3]{y} + \sqrt[5]{y}$

$\therefore y = 1$

so, $\left.\frac{dy}{dx}\right|_{y=1} = \frac{1}{\frac{1}{3} + \frac{1}{5}} = \frac{1}{\frac{8}{15}} = \boxed{\frac{15}{8}}$