Curve Sketching WS (solns)

1) $f(x) = \dfrac{3x^2 - 8x + 5}{x^3 - 1}$

disc

$x^3 - 1 = 0$

$\sqrt[3]{x^3} = \sqrt[3]{1}$

$x = 1$

No plus or minus with cube roots

label

Note: Only 2 types of disc w/ rational functions, removable and infinite.

Breaks or jump disc. are produced by piece-wise functions and Oscillations are produced by trig functions with problems dividing by 0.

Note: the directions don't say to find the limits @ infinite disc, only to label.

x = 1

$\lim_{x\to 1} \dfrac{3(1)^2 - 8(1) + 5}{x^3 - 1} = \dfrac{3 - 8 + 5}{0} = \dfrac{0}{0}$ indeterminate form

Factor: $\lim_{x\to 1} \dfrac{(3x-5)\cancel{(x-1)}}{\cancel{(x-1)}(x^2+x+1)} = \dfrac{3(1)-5}{1+1+1} = -\dfrac{2}{3}$

$\boxed{\therefore \text{ removable disc @ } x = 1}$

Zeros

top = 0

$3x^2 - 8x + 5 = 0$

$(3x-5)(x-1) = 0$

$x = \frac{5}{3} \quad x = 1$

A zero can't be the same as a disc.

$\boxed{\therefore \text{ only one zero @ } x = \frac{5}{3}}$

b/c x=1 is a disc

Horiz. Asymptotes

$$\lim_{x \to +\infty} f(x) \text{ and } \lim_{x \to -\infty} f(x)$$

polynomial — highest degree term or strongest term

$$\lim_{x \to +\infty} \frac{3x^2 - 8x + 5}{x^3 - 1} = \lim_{x \to +\infty} \frac{3x^2}{x^3} = \lim_{x \to +\infty} \frac{3}{x} = 0$$

polynomial — highest degree term

positive

$$\lim_{x \to -\infty} \frac{3x^2 - 8x + 5}{x^3 - 1} = \lim_{x \to -\infty} \frac{3x^2}{x^3} = \lim_{x \to +\infty} \frac{3x^2}{-x^3} = 0$$

negative

$\therefore$ Horizontal asymptote @ $y = 0$ in both directions

2) $$g(x) = \frac{\sqrt{16x^4 - 9}}{2x^2 - 5x - 7}$$

disc

$$2x^2 - 5x - 7 = 0$$

$$(2x - 7)(x + 1) = 0$$

disc @ $x = \frac{7}{2}$ $\quad x = -1$

label

$$\lim_{x \to -1} \frac{\sqrt{16(-1)^4 - 9}}{2(-1)^2 - 5(-1) - 7} = \frac{\sqrt{16 - 9}}{0} = \frac{\sqrt{7}}{0}$$

$\therefore$ infinite disc @ $x = -1$

$$\lim_{x \to \frac{7}{2}} \frac{\sqrt{16\left(\frac{7}{2}\right)^4 - 9}}{0} = \frac{\sqrt{\cancel{16}\left(\frac{7^4}{\cancel{16}}\right) - 9}}{0} = \frac{\sqrt{7^4 - 9}}{0}$$

$\therefore$ infinite disc @ $x = \frac{7}{2}$

Zeros $\left(\sqrt{16x^4-9}\right)^2 = (0)^2$

$$16x^4 - 9 = 0$$

$$16x^4 = 9$$

$$\sqrt[4]{x^4} = \frac{\sqrt[4]{9}}{\sqrt[4]{16}}$$

$$x = \pm \frac{\sqrt[4]{9}}{2}$$ (check with disc)

$\therefore$ Zeros @ $x = \pm \frac{\sqrt[4]{9}}{2}$

Horiz. Asymptotes

$$\lim_{x \to +\infty} \frac{\sqrt{16x^4 - 9}}{2x^2 - 5x - 7} = \lim_{x \to +\infty} \frac{\sqrt{16x^4}}{2x^2}$$

$$= \lim_{x \to +\infty} \frac{4x^2}{2x^2} = \frac{4}{2} = 2$$

$$\lim_{x \to -\infty} \frac{\sqrt{16x^4}}{2x^2} = \lim_{x \to -\infty} \frac{4x^2}{2x^2} = \lim_{x \to +\infty} \frac{4x^2}{2x^2} = 2$$

$+$ $+$

$\therefore$ horiz. asymptote @ $y = 2$ in both directions

3) $h(x) = \dfrac{4x^2 + 3x - 10}{e^x - 1}$

disc $e^x - 1 = 0$

$e^x = 1$ (in order to cancel out an e you take the natural log of both sides)

$\ln(e^x) = \ln(1)$ (memorize $\ln(1) = 0$)

$x = 0$

disc @ $x = 0$

label

$$\lim_{x\to 0} \frac{4x^2+3x-10}{e^x-1} = \frac{-10}{0}$$

$\therefore$ infinite disc @ $x=0$

Zeros

$$4x^2+3x-10=0$$

$$(4x-5)(x+2)=0$$

$$x=\frac{5}{4} \quad x=-2$$

$\therefore$ zeros @ $x=\frac{5}{4}$ and $x=-2$

Horizontal Asymptotes

$$\lim_{x\to+\infty} \frac{4x^2+3x-10}{e^x-1} = \lim_{x\to+\infty} \frac{4x^2}{e^x} = 0$$

strongest

$$\lim_{x\to-\infty} \frac{4x^2+3x-10}{e^x-1} = \lim_{x\to+\infty} \frac{4x^2+3x-10}{\frac{1}{e^x}-1}$$

pos

bottom of itself

strongest

goes to zero

$$= \lim_{x\to+\infty} \frac{+\infty}{0-1} = \frac{+\infty}{-1} = -\infty$$

$\therefore$ horiz asympt. @ $y=0$ only in the positive direction

4) $\phi(x) = \dfrac{\sin(x)-1}{x^2}$ on $[0, 2\pi]$

disc

$$x^2=0$$

$$x=0$$

disc @ $x=0$

label

$$\lim_{x\to 0} \frac{\sin(x)-1}{x^2} = \frac{\sin(0)-1}{0} = \frac{-1}{0}$$

$\therefore$ infinite disc @ $x=0$

zeros

$$\sin(x)-1=0$$

use a calculator with the zero command like we practiced in class.

$x = \frac{\pi}{2}$

$\therefore$ zero @ $x = \frac{\pi}{2}$

Horiz Asympt

$$\lim_{x \to +\infty} \frac{\sin(x) - 1}{x^2}$$

Note: $-1 \le \sin(x) \le 1 \to$ in words $\sin(x)$ is always between -1 and $+1$

So, $$\lim_{x \to +\infty} \frac{\sin(x) - 1}{x^2} = \frac{\#}{\infty} = 0$$

$$\lim_{x \to -\infty} \frac{\sin(x) - 1}{x^2} = \frac{\#}{\infty} = 0$$

$+$

$\therefore$ horiz. asymptote @ $y = 0$ in both directions