Piece-wise functions

2) $g(x) = \begin{cases} -x^2 - 2, & x < 1 \\ 2x - 5, & x \geq 1 \end{cases}$

<u>Discontinuities</u>:

*then → Check to see if function is cont. @ $x = 1$

$\lim_{x \to 1^-} -x^2 - 2 = -(1)^2 - 2 = -3$

$\lim_{x \to 1^+} 2x - 5 = 2(1) - 5 = -3$ $\quad \therefore \lim_{x \to 1} g(x) = -3$

$g(1) = 2(1) - 5 = -3$

So, $g(1) = \lim_{x \to 1} g(x) = -3$

$\therefore$ $g(x)$ is cont. @ $x = 1$

* Start here → Since $-x^2 - 2$ and $2x - 5$ are both polynomials and globally cont. then $g(x)$ is globally cont.

<u>Zeros</u>: Work both pieces separately with their respective domains.

1) $-x^2 - 2 = 0$ on $(-\infty, 1)$ or $x < 1$

$-x^2 = 2$

$x^2 = -2$

No soln. $\therefore$ No zeros on $(-\infty, 1)$ or $x < 1$

2) $2x - 5 = 0$ on $[1, +\infty)$ or $x \geq 1$

$2x = 5$

$x = \frac{5}{2}$ $\quad \therefore$ $g(x)$ has one zero @ $x = 2\frac{1}{2}$

<u>Horiz. asympt.</u>:

$\lim_{x \to -\infty} -x^2 - 2 = -\infty$

$\lim_{x \to +\infty} 2x - 5 = +\infty$

why don't we use $\lim_{x \to +\infty}$ with $-x^2 - 2$ and why don't we use $\lim_{x \to -\infty}$ with $2x - 5$.

$$g'(x) = \begin{cases} -2x , & x<1 \\ 2 , & x \geq 1 \end{cases}$$

$$\frac{-2x}{-2} = \frac{0}{-2} \qquad \text{and} \qquad 2 = 0$$

$x = 0$ No soln.

Critical pts @ $x=0$ and $x=1$ ← why $x=1$???

0		1	
-1	$\frac{1}{2}$		2
$-2(-1)$	$-2(\frac{1}{2})$		
$+$	$-$		$+$

$g(x)$ is inc on $(-\infty, 0], [1, +\infty)$ b/c g' is $+$.

$g(x)$ is dec. on $[0,1]$ b/c g' is neg.

$g(x)$ has a local min @ $x=1$ b/c g' changes from neg to pos.

$g(x)$ has <u>no</u> Absolute Extrema ← why???

$$g''(x) = \begin{cases} -2 , & x<1 \\ 0 , & x \geq 1 \end{cases}$$

$-2 = 0$ and $0 = 0$

No soln No soln.

Possible inflection pt. @ $x=1$

1	
0	2
$-$	0

$\therefore$ $g(x)$ is concave down on $(-\infty, 1)$ b/c g'' is $-$ and $g(x)$ is never concave up.

$g(x)$ has no inflection pts.