Quiz Prep. (another example)

see graph

2) $f(x) = (x^2-9)^{2/3}$

Zeros, discontinuities: original function

$(x^2-9)^{2/3} = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$x = \pm 3$ ∴ zeros (x-intercepts) @ $x = -3$ and $x = 3$

$f(x) = (x^2-9)^{2/3}$ has no discontinuities

Critical Points / Increasing and Decreasing / Extrema: 1st der.

$$f'(x) = \frac{2}{3}(x^2-9)^{-1/3}(2x) = \frac{4x}{3(x^2-9)^{1/3}} = 0$$

$4x = 0$

$x = 0$ critical point $\left(f'(x) = 0\right)$

Also, $3(x^2-9)^{1/3} = 0$

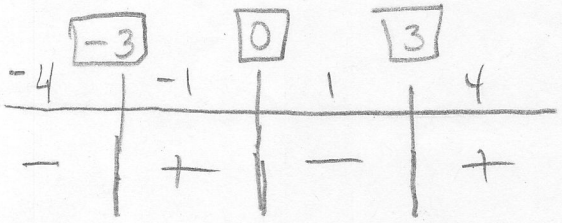
$(x^2-9)^{1/3} = 0$

$x^2 - 9 = 0$

$x^2 = 9$

$\left.\begin{matrix}\\\\\\\\\end{matrix}\right\}\left(f'(x) = \text{undefined}\right)$

$x = \pm 3$ critical points, ∴ 3 critical points @ $x = -3, x = 0, x = 3$



critical pts: $(-3, 0), (0, \sqrt[3]{-9^2}), (3, 0)$

$(0, -4.327)$

∴ $f(x)$ is inc on $[-3, 0]$ and $[3, +\infty)$ b/c $f'(x)$ is pos and $f(x)$ is dec. on $(-\infty, -3], [0, 3]$ b/c $f'(x)$ is neg.

∴ local maximum @ $x = 0$ b/c $f'(x)$ changes from pos to neg, and local minimums @ $x = -3$ and $x = 3$ b/c $f'(x)$ changes from neg to pos.

Quiz Prep (another example)

Inflection points / Concavity : 2nd der.

Rewrite $f'(x)$

$$f'(x) = \left(\frac{4}{3}x\right)(x^2-9)^{-1/3}$$ product rule

Note: $\frac{4}{3}$ is just a coefficient

$$f''(x) = \frac{4}{3}(x^2-9)^{-1/3} + \left(-\frac{1}{3}\right)(x^2-9)^{-4/3}(2x)\left(\frac{4}{3}x\right)$$

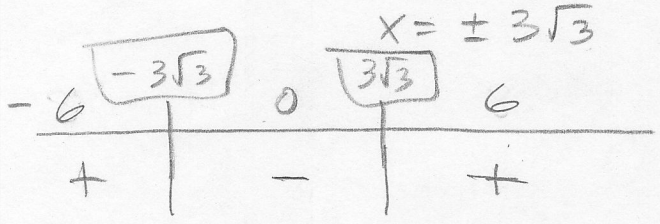
$$f''(x) = \frac{4}{3(x^2-9)^{1/3}} - \frac{8x^2}{9(x^2-9)^{4/3}}$$

common denominator - multiply 1st fraction by $\frac{3(x^2-9)}{3(x^2-9)}$

$$f''(x) = \frac{12(x^2-9)}{9(x^2-9)^{4/3}} - \frac{8x^2}{9(x^2-9)^{4/3}} = \frac{12x^2-108-8x^2}{9(x^2-9)^{4/3}}$$

$$f''(x) = \frac{4x^2-108}{9(x^2-9)^{4/3}} = 0$$

$$4x^2-108=0$$
$$4x^2=108$$
$$x^2=27$$
$$x=\pm 3\sqrt{3}$$



∴ $f(x)$ is concave up on $(-\infty, -3\sqrt{3})$ and $(3\sqrt{3}, +\infty)$ b/c $f''(x)$ is positive and $f(x)$ is concave down on $(-3\sqrt{3}, 3\sqrt{3})$ b/c $f''(x)$ is negative.

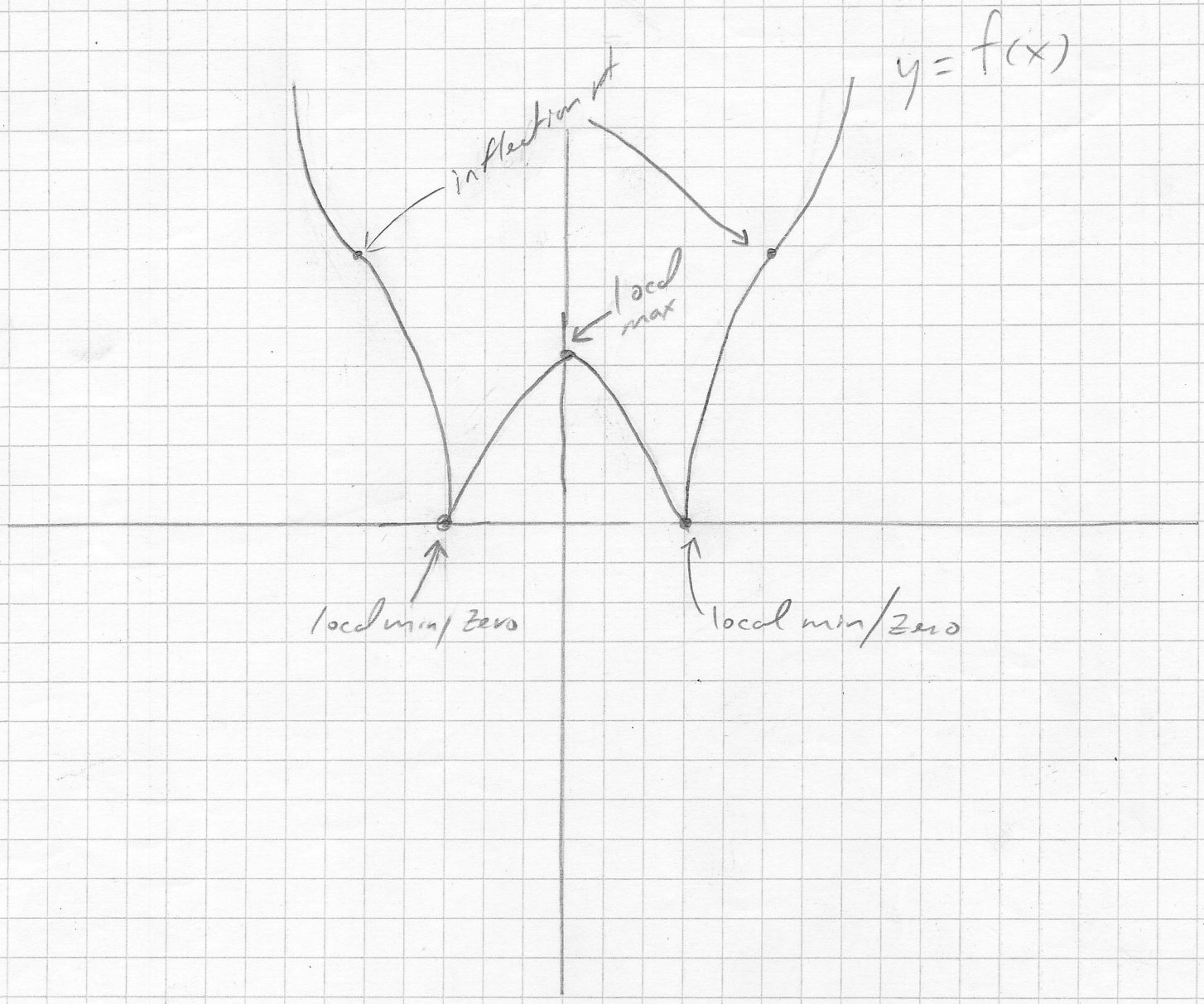
∴ the points of inflection are @ $x=-3\sqrt{3}$ and $x=3\sqrt{3}$

pts of inflection: $(-3\sqrt{3}, \sqrt[3]{18^2})$, $(3\sqrt{3}, \sqrt[3]{18^2})$

$(-5.19, 6.45)$ $(5.19, 6.45)$

Quiz prep (another example)

Using the information to sketch a graph.



Steps:

1. Plot zeros (x-intercepts), extrema and inflection pts.
2. Increasing $[-3,0]$, $[3,+\infty)$ and decreasing $(-\infty,-3]$, $[0,3]$
3. Concave up $(-\infty,-3\sqrt{3})$ and $(3\sqrt{3},+\infty)$ and concave down on $(-3\sqrt{3},3\sqrt{3})$
4. Sketch graph keeping in mind any discontinuities.

Compare to actual graph