

$$7) f(x) = \frac{3x}{x+2} \text{ on } (-\infty, +\infty)$$

$$f'(x) = \frac{3(x+2) - (1)(3x)}{(x+2)^2} = \frac{3x+6-3x}{(x+2)^2} = \frac{6}{(x+2)^2} = 0$$

No soln.

$$f'(x) \text{ undef: } (x+2)^2 = 0$$

$$x+2 = 0$$

$$x = -2$$

, however not a critical pt. b/c not in the domain of  $f(x)$  (original).

pick any #

$$\rightarrow 0$$

+

Always increasing

$\therefore$  No local max or min.

$\therefore$  No Absolute Max or Min

(Note:  $f(x)$  has a vertical asymptote @  $x = -2$ )

$$8) f(x) = \frac{2x^2}{e^x} \text{ on } (-\infty, +\infty)$$

$$f'(x) = \frac{4xe^x - e^x(2x^2)}{e^{2x}} = \frac{2xe^x(2-x)}{e^{2x}} = \frac{2x(2-x)}{e^x} = 0$$

$$\begin{aligned} 2x &= 0 \\ x &= 0 \end{aligned}$$

$$\begin{aligned} 2-x &= 0 \\ x &= 2 \end{aligned}$$

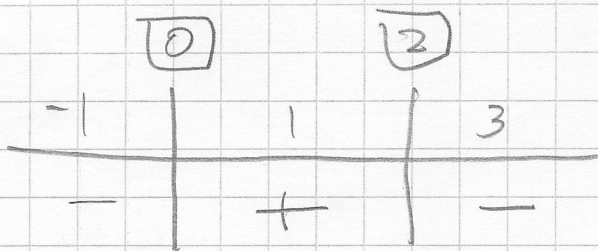
$$f'(x) \text{ undef: } e^x = 0$$

$$\ln e^x = \ln 0$$

DNE

No soln.

$\therefore$  only 2 critical pts @  $x=0, x=2$



$\therefore$  local min @  $x=0$  and local max @  $x=2$

Test for Absolute Max Min

$$\lim_{x \rightarrow +\infty} f(x) = \frac{2(+\infty)}{e^{\infty}} = \frac{+\infty}{+\infty} \text{ indeterminate form}$$

Using L'Hopital's Rule:  $\lim_{x \rightarrow +\infty} \frac{2x^2}{e^x} = \lim_{x \rightarrow +\infty} \frac{4x}{e^x} = \frac{+\infty}{+\infty}$

derivative of top/bottom separately

Use L'Hopital's Rule Again

$$\lim_{x \rightarrow +\infty} \frac{2x^2}{e^x} = \frac{4}{e^x} = \frac{4}{\infty} = 0 \rightarrow f(x) \text{ has horiz. asymptote @ } y=0$$

$$\lim_{x \rightarrow -\infty} \frac{2x^2}{e^x} = \frac{2(-\infty)^2}{e^{-\infty}} = 2(+\infty)e^{\infty} = +\infty \rightarrow \text{No Absolute Max.}$$

$$f(0) = 0$$

$\therefore$  Abs Min @  $x=0$  w/ min value of 0.

9)  $f(x) = 9x^3 - 6x^2 + 12x - 8$

Zeros (x-intercepts):  $(9x^3 - 6x^2) + 12x - 8 = 0$

$$3x^2(3x-2) + 4(3x-2) = 0$$

$$(3x-2)(3x^2+4) = 0$$

$$3x-2=0 \quad 3x^2+4=0$$

$$x = \frac{2}{3} \quad \sqrt{x^2} = \sqrt{\frac{-4}{3}} \text{ - imaginary soln.}$$

discontinuities:  $f(x)$  is a polynomial  $\rightarrow$  No disc.

1st Derivative:  $f'(x) = 27x^2 - 12x + 12 = 0$

critical points:  $f'(x) = 0$  and  $f'(x)$  undef.



$$f'(x) = 0 : 27x^2 - 27x + 12 = 0$$

$$3(9x^2 - 9x + 4) = 0$$

$$9x^2 - 9x + 4 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$b^2 - 4ac = (-9)^2 - 4(9)(4)$$

$$= 81 - 144 = (-)$$

means No soln.

$f'(x)$  undef:  $f'(x)$  is a polynomial  $\rightarrow$  Always continuous

$\therefore$  No critical points  $\rightarrow$  No local extrema

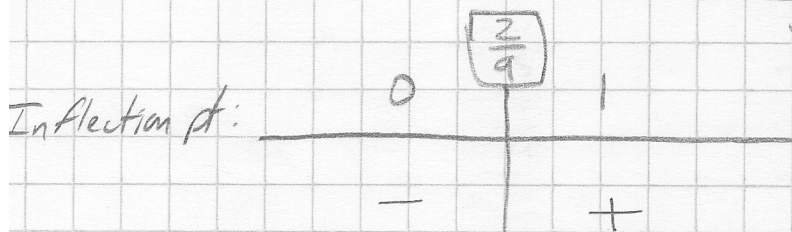
Inc/Dec:  $\frac{0}{+}$   $\leftarrow$  pick any #

$\therefore f(x)$  is increasing on  $(-\infty, +\infty)$  b/c  $f'(x) = +$

2nd Derivative:  $f''(x) = 54x - 12$

Concave up/down:  $54x - 12 = 0$

$$x = \frac{12}{54} = \frac{2}{9}$$



$\therefore$  Concave down on  $(-\infty, \frac{2}{9})$  b/c  $f''(x)$  is neg.  
and Concave up on  $(\frac{2}{9}, +\infty)$  b/c  $f''(x)$  is pos.  
 $\therefore$  inflection pt. @  $x = \frac{2}{9}$

10)  $f(x) = \frac{x^2 - 4}{x^2 - 25}$

Zeros (x-int):  $\frac{x^2 - 4}{x^2 - 25} = 0$

$$x^2 - 4 = 0$$

$$x = \pm 2$$

Zeros @  $x = -2$  and  $x = 2$

Discontinuities:  $f(x) = \text{undefined}$

$$f(x) = \frac{x^2 - 4}{x^2 - 25} = \text{undefined}$$

$$x^2 - 25 = 0$$

$x = \pm 5$  disc. @  $x = -5$  and  $x = 5$  but, what type are they?

Plug into original function to find out what type of disc.

$$f(-5) = \frac{(-5)^2 - 4}{(-5)^2 - 25} = \frac{21}{0} \quad \left\{ \begin{array}{l} \text{means vertical asymptote} \\ \text{(infinite disc) @ } x = -5. \end{array} \right.$$

$$f(5) = \frac{5^2 - 4}{5^2 - 25} = \frac{21}{0} \quad \left\{ \begin{array}{l} \text{also means vertical asymptote} \\ \text{(infinite disc.) @ } x = 5 \end{array} \right.$$

(Note: Remember  $\frac{0}{0}$  means hole or removable disc.)

1st derivative:  $f'(x) = \frac{2x(x^2 - 25) - (2x)(x^2 - 4)}{(x^2 - 25)^2}$

$$f'(x) = \frac{2x^3 - 50x - 2x^3 + 8x}{(x^2 - 25)^2} = \frac{-42x}{(x^2 - 25)^2}$$

critical points:  $f'(x) = 0$  and  $f'(x) = \text{undef}$

$$f'(x) = 0: -42x = 0$$
$$x = 0$$

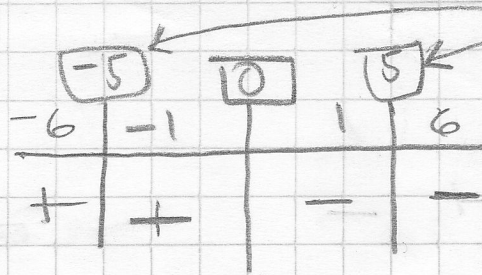
$$f'(x) = \text{undef}: (x^2 - 25)^2 = 0$$
$$x^2 = 25$$

$x = \pm 5$  but not critical pts b/c not in the domain of  $f(x)$ . (Remember there are vertical asymp. @  $x = \pm 5$  and you can't take the der. @ a discontinuity.)

$\therefore$  only 1 critical pt @  $x = 0$ .



Inc/Dec:



Don't need these values b/c they are not critical pts (notice, no sign change)

Local Extrema:

∴ inc on  $(-\infty, 0]$  b/c  $f'(x)$  is pos. and dec on  $[0, +\infty)$  b/c  $f'(x)$  is neg.

∴ local max @  $x=0$  with a value of  $f(0) = \frac{4}{25}$

2nd derivative:  $f''(x) = \frac{-42(x^2-25)^2 - (2)(x^2-25)(2x)(-42x)}{(x^2-25)^4}$

$f''(x) = \frac{-42(x^4 - 50x^2 + 625) - (4x^2)(x^2-25)(-42)}{(x^2-25)^4}$

$f''(x) = \frac{-42(x^4 - 50x^2 + 625 - 4x^4 + 100x^2)}{(x^2-25)^4}$

$f''(x) = \frac{-42(-3x^4 + 50x^2 + 625)}{(x^2-25)^4} = \frac{42(3x^4 - 50x^2 - 625)}{(x^2-25)^4}$

$\frac{42(3x^4 - 50x^2 - 625)}{(x^2-25)^4} = 0$

$42(3x^4 - 50x^2 - 625) = 0$

$3x^4 - 50x^2 - 625 = 0$

$(3x^2 + 25)(x^2 - 25) = 0$

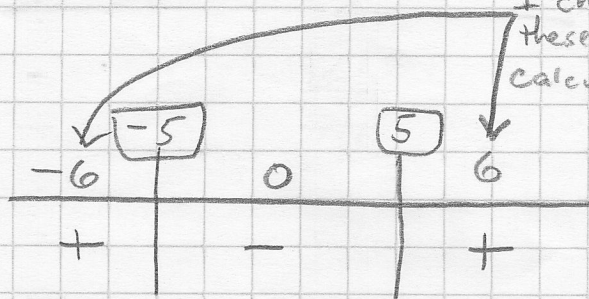
$3x^2 + 25 = 0$

$x^2 - 25 = 0$

$\sqrt{x^2} = \sqrt{-\frac{25}{3}}$

$x = \pm 5$

No soln.



I checked these w/a calculator

Notice the two -42 on the left and right of the outside minus sign. Can factor + out.

∴  $f(x)$  is concave up on  $(-\infty, -5)$  and  $(5, +\infty)$  b/c  $f''(x)$  is (+) and  $f(x)$  is concave down on  $(-5, 5)$  b/c  $f''(x)$  is (-).

However,  $x = -5$  and  $x = 5$  are not inflection pts b/c inflection pts have to be in the domain of the original function, and remember there are infinite disc. @  $x = \pm 5$ . ∴ No inflection pts.