7) $f(x) = \frac{3x}{x+2}$ on $(-\infty, +\infty)$

$$f'(x) = \frac{3(x+2) - (1)(3x)}{(x+2)^2} = \frac{3x+6-3x}{(x+2)^2} = \frac{6}{(x+2)^2} = 0$$

No soln.

$f'(x)$ undef: $(x+2)^2 = 0$

$x + 2 = 0$

$x = -2$, however not a critical pt. b/c not in the domain of $f(x)$ (original).

pick any # $\longrightarrow 0$

$+$

Always increasing

$\therefore$ No local max or min

$\therefore$ No Absolute Max or Min

(Note: $f(x)$ has a vertical asymptote @ $x = -2$)

8) $f(x) = \frac{2x^2}{e^x}$ on $(-\infty, +\infty)$

$$f'(x) = \frac{4xe^x - e^x(2x^2)}{e^{2x}} = \frac{2x\cancel{e^x}(2-x)}{e^{2x}} = \frac{2x(2-x)}{e^x} = 0$$

$2x = 0$ $\quad$ $2 - x = 0$

$x = 0$ $\quad$ $x = 2$

$f'(x)$ undef: $e^x = 0$

$\ln e^x = \ln 0$

DNE

No soln.

$\therefore$ only 2 critical pts @ $x=0$, $x=2$

	$\boxed{0}$		$\boxed{2}$	
-1		1		3
$-$		$+$		$-$

$\therefore$ local min @ $x=0$ and local max @ $x=2$

Test for Absolute Max Min

$$\lim_{x\to+\infty} f(x) = \frac{2(+\infty)}{e^{\infty}} = \frac{+\infty}{+\infty} \quad \text{indeterminate form}$$

derivative of top/bottom separately

Using L'Hopital's Rule: $\displaystyle\lim_{x\to+\infty} \frac{2x^2}{e^x} = \lim_{x\to+\infty} \frac{4x}{e^x} = \frac{+\infty}{+\infty}$

Use L'Hopital's Rule Again

$$\lim_{x\to+\infty} \frac{2x^2}{e^x} = \frac{4}{e^x} = \frac{4}{\infty} = 0 \longrightarrow f(x) \text{ has horiz. asymptote @ } y=0$$

$$\lim_{x\to-\infty} \frac{2x^2}{e^x} = \frac{2(-\infty)^2}{e^{-\infty}} = 2(+\infty)e^{\infty} = +\infty \longrightarrow \text{No Absolute Max.}$$

$f(0) = 0$

$\therefore$ Abs Min @ $x=0$ w/ Min Value of 0.

9) $f(x) = 9x^3 - 6x^2 + 12x - 8$

zeros (x-intercepts): $(9x^3 - 6x^2)(+12x - 8) = 0$

$$3x^2(3x-2) + 4(3x-2) = 0$$
$$(3x-2)(3x^2+4) = 0$$
$$3x - 2 = 0 \qquad 3x^2 + 4 = 0$$
$$x = \frac{2}{3} \qquad \sqrt{x^2} = \sqrt{-\frac{4}{3}} \text{ — imaginary soln.}$$

discontinuities: $f(x)$ is a polynomial $\longrightarrow$ No disc.

1st Derivative: $f'(x) = 27x^2 - 12x + 12$

critical points: $f'(x) = 0$ and $f'(x)$ undef.

$f'(x) = 0$: $27x^2 - 27x + 12 = 0$

$3(9x^2 - 9x + 4) = 0$

$9x^2 - 9x + 4 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\quad b^2 - 4ac = (-9)^2 - 4(9)(4)$

$= 81 - 144 = (-)$

means No soln.

$f'(x)$ undef: $f'(x)$ is a polynomial $\rightarrow$ Always continuous

$\therefore$ $\boxed{\text{No critical points}} \longrightarrow \boxed{\text{No local extrema}}$

Inc/Dec: 0 ← pick any #

$+$

$\therefore$ $f(x)$ is $\boxed{\text{increasing on } (-\infty, +\infty) \text{ b/c } f'(x) = +}$

2nd Derivative: $f''(x) = 54x - 12$

Concave up/down: $54x - 12 = 0$

$x = \dfrac{12}{54} = \dfrac{2}{9}$

Inflection pt: $0 \quad \boxed{\tfrac{2}{9}} \quad 1$

$- \quad\quad +$

$\therefore$ Concave down on $(-\infty, \frac{2}{9})$ b/c $f''(x)$ is neg

and Concave up on $(\frac{2}{9}, +\infty)$ b/c $f''(x)$ is pos

$\therefore$ inflection pt. @ $x = \frac{2}{9}$

10) $f(x) = \dfrac{x^2 - 4}{x^2 - 25}$

zeros (x-int): $\dfrac{x^2 - 4}{x^2 - 25} = 0$

$x^2 - 4 = 0$

$x = \pm 2$ $\quad \boxed{\text{Zeros @ } x = -2 \text{ and } x = 2}$

Discontinuities: $f(x) = \text{undefined}$

$$f(x) = \frac{x^2-4}{x^2-25} = \text{undefined}$$

$$x^2 - 25 = 0$$
$$x = \pm 5$$ disc. @ $x = -5$ and $x = 5$ but, what type are they?

Plug into original function to find out what type of disc.

$$f(-5) = \frac{(-5)^2-4}{(-5)^2-25} = \frac{21}{0}$$ means vertical asymptote (infinite disc) @ $x = -5$.

$$f(5) = \frac{5^2-4}{5^2-25} = \frac{21}{0}$$ also means vertical asymptote (infinite disc.) @ $x = 5$

(Note: Remember $\frac{0}{0}$ means hole or removable disc.)

1st derivative: $$f'(x) = \frac{2x(x^2-25) - (2x)(x^2-4)}{(x^2-25)^2}$$

$$f'(x) = \frac{2x^3 - 50x - 2x^3 + 8x}{(x^2-25)^2} = \frac{-42x}{(x^2-25)^2}$$

Critical points: $f'(x) = 0$ and $f'(x) = \text{undef}$

$f'(x) = 0$: $-42x = 0$
$$x = 0$$

$f'(x) = \text{undef}$: $(x^2-25)^2 = 0$
$$x^2 = 25$$
$$x = \pm 5$$ but not critical pts b/c not in the domain of $f(x)$. (Remember there are vertical asymp. @ $x = \pm 5$ and you can't take the der. @ a discontinuity)

$\therefore$ only 1 critical pt @ $x = 0$.

Don't need these values b/c they are <u>not</u> critical pts (notice, no sign change)

Inc/Dec:

local Extrema:

	$\boxed{-5}$		$\boxed{0}$		$\boxed{5}$	
-6		-1		1		6
$+$		$+$		$-$		$-$

∴ inc on $(-\infty, 0]$ b/c $f'(x)$ is pos. and dec on $[0, +\infty)$ b/c $f'(x)$ is neg.

∴ local max @ $x = 0$ with a value of $f(0) = \frac{4}{25}$

2nd derivative: $f''(x) = \dfrac{-42(x^2-25)^2 - (2)(x^2-25)(2x)(-42x)}{(x^2-25)^4}$

Notice the two -42 on the left and right of the outside minus sign. Can factor it out.

$$f''(x) = \frac{-42(x^4 - 50x^2 + 625) - (4x^2)(x^2-25)(-42)}{(x^2-25)^4}$$

$$f''(x) = \frac{-42(x^4 - 50x^2 + 625 - 4x^4 + 100x^2)}{(x^2-25)^4}$$

$$f''(x) = \frac{-42(-3x^4 + 50x^2 + 625)}{(x^2-25)^4} = \frac{42(3x^4 - 50x^2 - 625)}{(x^2-25)^4}$$

$$\frac{42(3x^4 - 50x^2 - 625)}{(x^2-25)^4} = 0$$

$42(3x^4 - 50x^2 - 625) = 0$

$3x^4 - 50x^2 - 625 = 0$

$(3x^2 + 25)(x^2 - 25) = 0$

$3x^2 + 25 = 0$ $\quad$ $x^2 - 25 = 0$

$\sqrt{x^2} = \sqrt{-\frac{25}{3}}$ $\quad$ $x = \pm 5$

No soln.

I checked these w/a calculator

	$\boxed{-5}$		$\boxed{5}$	
-6		0		6
$+$		$-$		$+$

∴ $f(x)$ is concave up on $(-\infty, -5)$ and $(5, +\infty)$ b/c $f''(x)$ is $(+)$ and $f(x)$ is concave down on $(-5, 5)$ b/c $f''(x)$ is $(-)$.

However, $x = -5$ and $x = 5$ are not inflection pts b/c inflection pts have to be in the domain of the original function, and remember there are infinite disc. @ $x = \pm 5$.
∴ No inflection pts.