Critical Points + Extrema WS

1) $f(x) = 2x^4 + 27x$ on $[-3, 1]$

Extreme Value Th$^{\underline{m}}$ guarantees an Absolute Max and Min on a closed interval

$f'(x) = 8x^3 + 27 = 0$

$8x^3 = -27$

$x^3 = -\frac{27}{8}$

$x = \frac{\sqrt[3]{-27}}{\sqrt[3]{8}} = -\frac{3}{2}$

$\boxed{-\frac{3}{2}}$

-2	0
$-$	$+$

$\therefore$ local min @ $x = -\frac{3}{2}$ b/c $f(x)$ changes from decreasing to increasing

Test for Absolute Max/Min

$f(-\frac{3}{2}) = 2(-\frac{3}{2})^4 + 27(-\frac{3}{2}) = 2\left(\frac{81}{16}\right) - \frac{81}{2} = \frac{81}{8} - \frac{81 \cdot 4}{2 \cdot 4}$

$= \frac{81}{8} - \frac{324}{8} = \underline{\frac{-243}{8}}$

check endpts

$f(-3) = 2(-3)^4 + 27(-3) = 162 - 81 = \underline{81}$

$f(1) = 2(1)^4 + 27(1) = 2 + 27 = \underline{29}$

$\therefore$ The Absolute min is @ $x = -\frac{3}{2}$ and has a minimum value of $-\frac{243}{8}$.

The Absolute max is @ $x = -3$ and has a maximum value of 81.

(Note: graph $y = f(x)$ on graphing calculator)

2) $f(x) = \sqrt{3}\sin x - \cos x - 2x$ on $[-\pi, \pi]$

Extreme Value Th$^{\underline{m}}$

$f'(x) = \sqrt{3}\cos x + \sin x = 0$

$\sin x = -\sqrt{3}\cos x$

$\tan x = -\sqrt{3}$

$x = \frac{2\pi}{3}, \frac{5\pi}{3}$ ← too big / get to this angle going in the negative direction

$\therefore\ x = \frac{2\pi}{3}, \frac{-\pi}{3}$

$-\pi$	$\boxed{-\frac{\pi}{3}}$	0	$\boxed{\frac{2\pi}{3}}$	π
$-$		$+$		$-$

on $[-\pi, \pi]$

$\therefore$ local min. @ $x = -\frac{\pi}{3}$, local max @ $x = \frac{2\pi}{3}$
b/c $f'(x)$ changes from neg to pos @ $x = -\frac{\pi}{3}$
and $f'(x)$ changes from pos to neg @ $x = \frac{2\pi}{3}$.

Test for absolute max and min.

$$f\left(-\tfrac{\pi}{3}\right) = \sqrt{3}\sin\left(\tfrac{-\pi}{3}\right) - \cos\left(\tfrac{-\pi}{3}\right) - 2\pi$$
$$= \sqrt{3}\sin\left(\tfrac{5\pi}{3}\right) - \cos\left(\tfrac{5\pi}{3}\right) - 2\pi$$
$$= \sqrt{3}\left(-\sqrt{3}/2\right) - \tfrac{1}{2} - 2\pi = -\tfrac{3}{2} - \tfrac{1}{2} - 2\pi = -2 - 2\pi$$

$$f\left(\tfrac{2\pi}{3}\right) = \sqrt{3}\sin\left(\tfrac{2\pi}{3}\right) - \cos\left(\tfrac{2\pi}{3}\right) - 2\pi$$
$$= \sqrt{3}\left(\sqrt{3}/2\right) - \left(-\tfrac{1}{2}\right) - 2\pi = \tfrac{3}{2} + \tfrac{1}{2} - 2\pi = 2 - 2\pi$$

check endpts

$$f(-\pi) = \sqrt{3}\sin(-\pi) - \cos(-\pi) - 2\pi$$
$$= \sqrt{3}(0) - (-1) - 2\pi = 1 - 2\pi$$

$$f(\pi) = f(-\pi) = 1 - 2\pi$$

$\therefore$ Absolute max @ $x = \frac{2\pi}{3}$ w/ a max value of $2 - 2\pi$
Absolute min @ $x = -\frac{\pi}{3}$ w/ a min value of $-2 - 2\pi$

3) $f(x) = x^2 - 4x + 4$ on $(-\infty, +\infty)$

$$f'(x) = 2x - 4 = 0$$
$$x = 2$$

0	$\boxed{2}$	3
$-$		$+$

$\therefore$ local min @ $x = 2$ b/c $f(x)$
changes from dec to inc

Test for Absolute max and min.

check endpts.

$$\lim_{x\to-\infty} x^2-4x+4 = \lim_{x\to-\infty} x^2 = (-\infty)^2 = +\infty, \therefore \text{No Abs. max.}$$

$$\lim_{x\to+\infty} x^2-4x+4 = \lim_{x\to+\infty} x^2 = (+\infty)^2 = +\infty$$

$\therefore$ Absolute minimum @ $x = 2$ w/ a minimum value of $f(2) = 2^2-4(2)+4 = 0$.

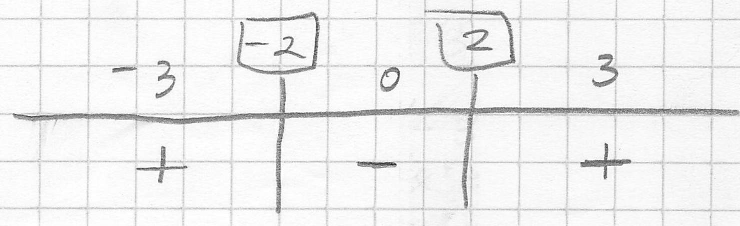
4) $f(x) = \frac{2x^5}{5} - 32x$ on $(-\infty, +\infty)$

$$f'(x) = 2x^4 - 32 = 0$$

$$2x^4 = 32$$

$$x^4 = 16$$

$$x = \pm\sqrt[4]{16} = \pm 2$$



$\therefore$ local max @ $x=-2$, local min @ $x = 2$ b/c $f'(x)$ changes from pos to neg. @ $x=-2$ and neg to pos @ $x = 2$.

Test for Absolute max and min.

$$\lim_{x\to-\infty} \frac{2x^5}{5} - 32x = \lim_{x\to-\infty} \frac{2x^5}{5} = \frac{2(-\infty)^5}{5} = -\infty$$

$\therefore$ No Abs. min.

$$\lim_{x\to+\infty} \frac{2x^5}{5} - 32x = \lim_{x\to+\infty} \frac{2x^5}{5} = \frac{2(+\infty)^5}{5} = +\infty$$

$\therefore$ No Abs. max.

No Absolute Max and No Absolute Min

5) $f(x) = \dfrac{e^{2x-1}}{2} - x$ on $(-\infty, +\infty)$

$$f'(x) = \frac{e^{2x-1}}{\cancel{2}}\cancel{(2)} - 1 = e^{2x-1} - 1 = 0$$

$$e^{2x-1} = 1$$

take the ln of both sides.

$$\ln e^{2x-1} = \ln 1$$
$$(2x-1)\ln e = 0$$
$$(2x-1)(1) = 0$$
$$2x - 1 = 0$$
$$x = \tfrac{1}{2}$$

$\boxed{\tfrac{1}{2}}$

0	1
$-$	$+$

$\therefore$ local min @ $x = \frac{1}{2}$ b/c $f(x)$ changes from dec. to inc.

Test for Absolute Max + Min

$$\lim_{x \to -\infty} \frac{e^{2x-1}}{2} - x = \frac{e^{-\infty}}{2} - (-\infty) = \frac{1}{2e^{\infty}} + \infty = 0 + \infty$$
$$= +\infty$$

$\therefore$ No Absolute Max.

$$\lim_{x \to +\infty} \frac{e^{2x-1}}{2} - x = \frac{e^{+\infty}}{2} - \infty = \infty - \infty$$

indeterminette form

Question: which infinity is "stronger?"

Answer: e^x is "stronger" than x

exponential function

polynomial

$$\therefore \lim_{x \to +\infty} \frac{e^{2x-1}}{2} - x = +\infty$$

$\therefore$ Absolute min @ $x = \frac{1}{2}$ w/ a minimum value of $f(\frac{1}{2}) = \dfrac{e^0}{2} - \dfrac{1}{2} = 0$

6) $f(x) = \sqrt[3]{x^2-25}$ on $(-\infty, +\infty)$
$= (x^2-25)^{1/3}$

$$f'(x) = \frac{1}{3}(x^2-25)^{-2/3}(2x) = \frac{2x}{3(x^2-25)^{2/3}} = 0$$

Critical pts.

$f'(x) = 0$: $2x = 0$
$x = 0$ in the domain of $f(x)$

$f'(x)$ undefined: $3(x^2-25)^{2/3} = 0$
$(x^2-25)^{2/3} = 0$
$x^2 - 25 = 0$
$x^2 = 25$
$x = \pm 5$ in the domain of $f(x)$

$\therefore$ 3 critical pts @ $x = -5, 0, 5$

	$\boxed{-5}$		$\boxed{0}$		$\boxed{5}$	
-6	|	-1	|	1	|	6
$-$	|	$-$	|	$+$	|	$+$

$\therefore$ local min. @ $x = 0$

Test for Abs. Max and Min

Only 3 possibilities for Absolute Max and Min are @ $x = 0$ (local min) or @ the endpoints ($-\infty$ and $+\infty$)
Can't have Abs. Max or Min @ $-\infty$ or $+\infty$,
$\therefore$ The only possibility is that $x = 0$ is the location of our Absolute Min.

$$\lim_{x \to +\infty} f(x) = \sqrt[3]{(+\infty)^2 - 25} = +\infty$$

$$\lim_{x \to -\infty} f(x) = \sqrt[3]{(-\infty)^2 - 25} = +\infty$$

$\therefore$ Absolute Min @ $x = 0$ w/ a Minimum value of $f(0) = \sqrt[3]{-25}$