# Continuity WS

1) 1) $\lim_{x \to 2} f(x) = 2(2)^2 - 2 + 1 = 7$ exists ✓

2) $f(2) = 2(2)^2 - 2 + 1 = 7$ exists ✓

3) $\lim_{x \to 2} f(x) = 7 = f(2)$

$\therefore f(x)$ is cont. @ $x = 2$

2) 1) $\lim_{x \to 1} g(x) = \frac{1-1}{1} = \frac{0}{1} = 0$ exists ✓

2) $g(1) = \frac{1-1}{1} = 0$ exists ✓

3) $\lim_{x \to 1} g(x) = 0 = g(1)$

$\therefore g(x)$ is cont. @ $x = 1$

3) 1) $\lim_{x \to 0} h(x) = \sin(0) + \tan(0) = 0 + 0 = 0$ exists ✓ (→ use a calculator (radian mode))

2) $h(0) = \sin(0) + \tan(0) = 0 + 0 = 0$ exists ✓

3) $\lim_{x \to 0} h(x) = 0 = h(0)$

$\therefore h(x)$ is cont. @ $x = 0$

4) $p(t) = \dfrac{4t + 10}{t^2 - 2t - 15}$

disc| $t^2 - 2t - 15 = 0$

$(t-5)(t+3) = 0$

disc @ $t = 5$ $\quad t = -3$

$\boxed{x=5}$ 1) $\lim_{x\to 5} \frac{4t+10}{t^2-2t-15} = \frac{30}{0}$

$\lim_{x\to 5^-} \frac{2(2t+5)}{(t-5)(t+3)} = \frac{+}{(-)(+)} = \frac{+}{-} = -\infty$

$\lim_{x\to 5^+} \frac{2(2t+5)}{(t-5)(t+3)} = \frac{+}{(+)(+)} = \frac{+}{+} = +\infty$

$\lim_{x\to 5} p(t)$ DNE ✗

$\therefore$ $p(t)$ is not cont. @ $x=5$.

$\boxed{x=-3}$ You don't have to do this in order.

2) $p(-3) = \frac{4(-3)+10}{(-3)^2-2(-3)-15} = \frac{-2}{0}$ undefined or DNE ✗

$\therefore$ $p(t)$ is not continuous @ $x=-3$.

5) $\phi(x) = \sin(x) + \tan(x) = \frac{\sin(x)}{1} + \frac{\sin(x)}{\cos(x)} = \frac{\sin(x)\cos(x)}{\cos(x)} + \frac{\sin(x)}{\cos(x)}$

$= \frac{\sin(x)\cos(x) + \sin(x)}{\cos(x)}$

disc: $\cos(x) = 0$ this equation has an infinite number of solutions. For that reason use an initial domain of $[0, 2\pi]$ ← x-values

$\cos(x) = 0$ @ $x = \frac{\pi}{2}, \frac{3\pi}{2}$ (used a calculator)

$\boxed{\text{disc @ } x = \frac{\pi}{2}, \frac{3\pi}{2}}$

$\boxed{x=\frac{\pi}{2}}$ 2) $\phi(\frac{\pi}{2}) = \frac{\sin(\frac{\pi}{2})\cos(\frac{\pi}{2}) + \sin(\frac{\pi}{2})}{\cos(\frac{\pi}{2})} = \frac{0+1}{0} = \frac{1}{0}$ undef ✗

$\therefore$ $\phi(x)$ is not cont. @ $x = \frac{\pi}{2}$

$\boxed{x=\frac{3\pi}{2}}$ 2) $\phi(\frac{3\pi}{2}) = \frac{\sin(\frac{3\pi}{2})\cos(\frac{3\pi}{2}) + \sin(\frac{3\pi}{2})}{\cos(\frac{3\pi}{2})} = \frac{0+1}{0} = \frac{-1}{0}$ undef ✗

$\therefore \Phi(x)$ is not cont. @ $x = \frac{3\pi}{2}$

6) $\lim_{x \to 3} \frac{x^3 - 5x + 4}{x^2 - 2} = \frac{(3)^3 - 5(3) + 4}{(3)^2 - 2} = \frac{16}{7}$

7) $\lim_{x \to 1} \frac{x^2 - 1}{1 - x} = \frac{0}{0}$

$\lim_{x \to 1} \frac{(x+1)(x-1)}{1-x} = \frac{(x+1)(x-1)}{-1(x-1)} = \frac{2}{-1} = -2$

8) $\lim_{x \to 0} \frac{2x}{3 - \sqrt{x+9}} = \frac{0}{0}$

$\lim_{x \to 0} \frac{2x}{3 - \sqrt{x+9}} \cdot \frac{3 + \sqrt{x+9}}{3 + \sqrt{x+9}} = \frac{(2x)(3 + \sqrt{x+9})}{9 - (x+9)} = \frac{2x(3 + \sqrt{x+9})}{9 - x - 9}$

$= \lim_{x \to 0} \frac{2x(3 + \sqrt{x+9})}{-x} = \frac{2(3 + \sqrt{9})}{-1} = -12$

9) a) $\lim_{x \to 0^-} \frac{\sin(4x)}{x} = \frac{4}{1} = 4$ ← memorize $\frac{\sin(ax)}{bx} = \frac{a}{b}$ as $x \to 0$

b) $\lim_{x \to 0^+} x^2 + 4 = 4$

c) $\lim_{x \to 0} f(x) = 4$

d) $\lim_{x \to 2^-} x^2 + 4 = 4 + 4 = 8$

e) $\lim_{x \to 2^+} \frac{x^2 - 4}{x^2 - 4x + 4} = \frac{0}{0}$

$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)(x-2)} = \frac{4}{0}$ ← infinit disc

$= \lim_{x \to 2^+} \frac{x+2}{x-2} = \frac{+}{+} = +\infty$

f) $\lim_{x \to 2} f(x)$ DNE

g) $f(0) = 0^2 + 4 = 4$

h) $f(2) = \frac{2^2 - 4}{2^2 - 4(2) + 4} = \frac{0}{0}$ undef. or DNE

10) $\lim_{x \to 1^-} x^2 + x + p = \lim_{x \to 1^+} x^3$

$1^2 + 1 + p = 1^3$

$2 + p = 1$

$p = -1$

Note: one variable so you only need one equation.