p. 417 #5-17 odd

5) a) $v(t) = 3t^2 - 2t, \quad s(0) = 1$

$s(t) = \int v(t) = \int 3t^2 - 2t = t^3 - t^2 + c$

$s(0) = 0^3 - 0^2 + c = 1 \implies c = 1$

$s(t) = t^3 - t^2 + 1$

b) $a(t) = 3\sin 3t, \quad v(0) = 3, \quad s(0) = 3$

$v(t) = \int a(t) = \int 3\sin 3t = -\cos 3t + c$

$v(0) = -\cos 3(0) + c = 3$

$= -1 + c = 3$

$c = 4$

$v(t) = -\cos 3t + 4$

$s(t) = \int v(t) = \int -\cos 3t + 4 = -\frac{\sin 3t}{3} + 4t + c$

$s(0) = -\frac{\sin(0)}{3} + 4(0) + c = 3$

$c = 3$

$\therefore s(t) = -\frac{\sin 3t}{3} + 4t + 3$

7) a) $v(t) = 3t + 1 \quad s(2) = 4$

$s(t) = \int v(t) = \int 3t + 1 = \frac{3t^2}{2} + t + c$

$s(2) = \frac{3(2)^2}{2} + 2 + c = 4 \implies c = -4$

2

$s(t) = \frac{3t^2}{2} + t - 4$

b) $a(t) = t^{-2}$, $v(1) = 0$, $s(1) = 2$

$v(t) = \int t^{-2} dt = -\frac{1}{t} + c$

$v(1) = -\frac{1}{1} + c = 0 \Rightarrow c = 1$

$v(t) = -\frac{1}{t} + 1$

$s(t) = \int v(t) = \int -\frac{1}{t} + 1 \, dt = -\ln|t| + t + c$

$s(1) = -\ln 1 + 1 + c = 2 \Rightarrow c = 1$

$s(t) = -\ln|t| + t + 1$

9) a) $v(t) = \sin t$, $0 \le t \le \frac{\pi}{2}$

displacement $= \int_0^{\frac{\pi}{2}} v(t) = \int_0^{\frac{\pi}{2}} \sin t \, dt = -\cos t \Big|_0^{\frac{\pi}{2}} = \boxed{1 \text{ m}}$

distance $= \int_0^{\frac{\pi}{2}} |v(t)|$

$v(t) = \sin t = 0$ @ $t = 0, \pi$ ← too big

$\therefore$ sint doesnot cross the x-axis between 0 and $\frac{\pi}{2}$.

$\therefore$ Area is above x-axis

distance $= \int_0^{\frac{\pi}{2}} v(t) = \int_0^{\frac{\pi}{2}} \sin t = \boxed{1 \text{ m}}$

b) $v(t) = \cos t$, $\frac{\pi}{2} \le t \le 2\pi$

displacement $= \int_{\frac{\pi}{2}}^{2\pi} \cos t \, dt = \sin t \Big|_{\frac{\pi}{2}}^{2\pi} = \sin 2\pi - \sin\frac{\pi}{2} = \boxed{-1 \text{ m}}$

$$\text{distance} = \int_{\frac{\pi}{2}}^{2\pi} |v(t)|$$

$$v(t) = \cos t = 0$$

$$\therefore t = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\text{distance} = \underbrace{\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos t}_{\text{negative}} + \underbrace{\int_{\frac{3\pi}{2}}^{2\pi} \cos t}_{\text{positive}} = -\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cos t\,dt + \int_{\frac{3\pi}{2}}^{2\pi} \cos t\,dt$$

$$\Rightarrow d = \sin t \Big|_{\frac{3\pi}{2}}^{\frac{\pi}{2}} + \sin t \Big|_{\frac{3\pi}{2}}^{2\pi} = (1-(-1)) + (0-(-1)) = \boxed{3m}$$

11) a) $v(t) = t^3 - 3t^2 + 2t, \quad 0 \leq t \leq 3$

$$\text{displacement} = \int_0^3 t^3 - 3t^2 + 2t\,dt$$

$$= \frac{t^4}{4} - t^3 + t^2 \Big|_0^3 = \frac{81}{4} - 27 + 9 = \frac{81}{4} - 18 = \boxed{\frac{9}{4}}$$

$$\text{distance} = \int_0^3 |v(t)|\,dt$$

$$t^3 - 3t^2 + 2t = 0$$

$$t(t^2 - 3t + 2) = 0$$

$$t(t-2)(t-1) = 0$$

$$t = 0,\ t = 2,\ t = 1$$

$$\text{distance} = \underbrace{\int_0^1 t^3 - 3t^2 + 2t}_{\text{positive}} + \underbrace{\int_1^2 t^3 - 3t^2 + 2t}_{\text{negative}} + \underbrace{\int_2^3 t^3 - 3t^2 + 2t}_{\text{positive}}$$

$$\text{distance} = \int_0^1 t^3-3t^2+2t - \int_1^2 t^3-3t^2+2t + \int_2^3 t^3-3t^2+2t$$

$$= \frac{t^4}{4} - t^3 + t^2 \Big|_0^1 + \frac{t^4}{4} - t^3 + t^2 \Big|_2^1 + \frac{t^4}{4} - t^3 + t^2 \Big|_2^3$$

$$= \frac{1}{4} + \left(\frac{1}{4} - (4-8+4)\right) + \left(\frac{81}{4} - 27 + 9\right) = \boxed{\frac{11}{4}}$$

b) $\text{Displacement} = \int_0^3 \sqrt{t} - 2\,dt = \frac{2}{3}t^{3/2} - 2t \Big|_0^3 = \frac{2\sqrt{3^3}}{3} - 6$

$$= \frac{2 \cdot \cancel{3}\sqrt{3}}{\cancel{3}} - 6 = \boxed{2\sqrt{3} - 6}$$

c) $\text{Distance} = \int_0^3 \left|\sqrt{t} - 2\right| dt$

intersects x-axis @ : $\sqrt{t} - 2 = 0$

$$(\sqrt{t})^2 = (2)^2$$

$t = 4 \leftarrow$ not in between 0 and 3 so disregard it

$\text{Dist.} = \int_0^3 \underset{\substack{\uparrow \\ \text{negative}}}{\sqrt{t} - 2}\, dt = -\int_0^3 \sqrt{t} - 2\,dt$

$$= \frac{2}{3}t^{3/2} - 2t \Big|_3^0 = \boxed{6 - 2\sqrt{3}}$$

13) $a(t) = 3, \quad V_0 = -1, \qquad 0 \le t \le 2$

$$V(t) = \int a(t)\,dt = \int 3\,dt = 3t + C$$

$$V(0) = 3(0) + C = -1$$

$$C = -1$$

$$V(t) = 3t - 1$$

$$\text{Displacement} = \int_0^2 3t-1\,dt = \frac{3t^2}{2} - t\Big|_0^2 = \boxed{4}$$

$$\text{Distance} = \int_0^2 |v(t)|\,dt = \int_0^2 |3t-1|\,dt$$

intersects x-axis @ : $3t-1=0$

$3t=1$

$t=\frac{1}{3}$

$$\text{Dist.} = \underbrace{\int_0^{\frac{1}{3}} 3t-1\,dt}_{\text{neg}} + \underbrace{\int_{\frac{1}{3}}^2 3t-1\,dt}_{\text{pos.}} = -\int_0^{\frac{1}{3}} 3t-1\,dt + \int_{\frac{1}{3}}^2 3t-1\,dt$$

$$\text{Dist.} = \frac{3t^2}{2} - t\Big|_{\frac{1}{3}}^0 + \frac{3t^2}{2} - t\Big|_{\frac{1}{3}}^2 = \frac{1}{6} + \left(4-\left(-\frac{1}{6}\right)\right) = 4\frac{1}{3} = \boxed{\frac{13}{3}}$$

15) $a(t) = \frac{1}{\sqrt{3t+1}}$, $V_0 = \frac{4}{3}$, $1 \le t \le 5$

$$V(t) = \int a(t)\,dt = \int \frac{1}{\sqrt{3t+1}}\,dt = \int \frac{1}{\sqrt{u}}\left(\frac{du}{3}\right) = \frac{1}{3}\int u^{-1/2}\,du$$

$u = 3t+1$

$du = 3\,dt$

$dt = \frac{du}{3}$

$$= \frac{1}{3}\left(\frac{u^{1/2}}{\frac{1}{2}}\right) + C$$

$$= \frac{2}{3}\sqrt{u} + C$$

$$= \frac{2}{3}\sqrt{3t+1} + C$$

$$V(0) = \frac{2}{3}\sqrt{0+1} + C = \frac{4}{3} \Rightarrow C = \frac{2}{3}$$

$$V(t) = \frac{2\sqrt{3t+1}}{3} + \frac{2}{3} = \frac{2}{3}\left(\sqrt{3t+1} + 1\right)$$

$$\text{Displacement} = \frac{2}{3}\int_1^5 \sqrt{3t+1} + 1\,dt = \frac{2}{3}\left[\left(\frac{2}{9}\right)(3t+1)^{3/2} + t\Big|_1^5\right.$$

$u = 3t+1$

$du = 3\,dt$

$dt = \frac{du}{3}$

$$\text{Displacement} = \frac{2}{3}\left[\left(\frac{2}{9}(64)+5\right)-\left(\frac{2}{9}(8)+1\right)\right]$$

$$\text{Displacement} = \frac{2}{3}\left[\frac{128}{9}+5-\frac{16}{9}-1\right] = \frac{2}{3}\left(\frac{112}{9}+4\right) = \frac{2}{3}\left(\frac{148}{9}\right) = \boxed{\frac{296}{27}}$$

$$\text{Distance} = \int_1^5 |v(t)|\,dt = \int_1^5 \left|\sqrt{3t+1}+1\right|dt$$

$$\sqrt{3t+1}+1=0$$

$$\sqrt{3t+1}=-1$$

No solution

$$\text{Dist.} = \int_1^5 \underset{\uparrow \text{ pos}}{\sqrt{3t+1}+1} = \boxed{\frac{296}{27}}$$ (same as displacement)

17) a) $v=\sin\left(\frac{1}{2}\pi t\right)$, $s=0$ when $t=0$

$$s(t)=\text{position} = \int v = \int \sin\left(\tfrac{1}{2}\pi t\right) = \frac{-\cos\left(\frac{1}{2}\pi t\right)}{\frac{1}{2}\pi}+C = \frac{-2\cos\left(\frac{1}{2}\pi t\right)}{\pi}+C$$

$s=0$ when $t=0$

$$\frac{-2\cos\left(\frac{1}{2}\pi(0)\right)}{\pi}+C=0$$

$$-\frac{2}{\pi}+C=0$$

$C=\frac{2}{\pi}$ $\therefore$ position function $= s(t) = \frac{-2\cos\left(\frac{1}{2}\pi t\right)}{\pi}+\frac{2}{\pi}$

$$S(1) = \frac{-2\cos\left(\frac{\pi}{2}\right)}{\pi}+\frac{2}{\pi} = \boxed{\frac{2}{\pi}} \rightarrow \text{position @ } t=1$$

$V(1) = \sin\frac{\pi}{2} = \boxed{1}$ velocity @ $t=1$ speed $= |v(t)| = |1| = \boxed{1}$ speed @ $t=1$

$a(t) = \cos\left(\frac{\pi t}{2}\right)\left(\frac{\pi}{2}\right)$

$a(1) = \frac{\pi}{2}\cos\left(\frac{\pi}{2}\right) = \boxed{0} \rightarrow$ accleration @ $t=1$

b) $a = -3t$, $s = 1$, $v = 0$ when $t = 0$

$a(1) = -3(1) = -3 \rightarrow$ accleration @ $t=1$

$v(t) = \int a(t)\,dt = \int -3t\,dt = -\frac{3t^2}{2} + c$

$v(0) = -\frac{3(0)^2}{2} + c = 0 \implies c = 0$

$v(t) = -\frac{3t^2}{2}$

$v(1) = -\frac{3(1)^2}{2} = \boxed{-\frac{3}{2}} \rightarrow$ velocity @ $t=1$

speed $= |v(t)| = \left|-\frac{3}{2}\right| = \boxed{\frac{3}{2}} \rightarrow$ speed @ $t=1$

$s(t) = \int v(t)\,dt = \int -\frac{3t^2}{2} = -\frac{t^3}{2} + c$

$s(0) = \frac{0}{2} + c = 1 \implies c = 1$

$s(t) = -\frac{t^3}{2} + 1$

$s(1) = -\frac{1}{2} + 1 = \boxed{\frac{1}{2}}$