# Area under the Curve WS

1) $2-2x=0$

$x=1$

| -1 | | 1 | | 3 |
|---|---|---|---|---|
| | 0 | | 2 | |
| | + | | − | |

$$\int_{-1}^{1} 2-2x-0\,dx+\int_{1}^{3} 0-(2-2x)\,dx$$

changed to addition b/c limits changed

$$2x-x^2\Big|_{-1}^{1} + 2x-x^2\Big|_{3}^{1}$$

$$=\left(2(1)-1^2\right)-\left(2(-1)-(-1)^2\right)$$

$$=(2-1)-(-2-1)=1+3=4$$

$$\left(2(1)-(1)^2\right)-\left(2(3)-3^2\right)$$

$$1-(6-9)=4$$

$$4+4=8$$

2) $x^2-4=0$

$x=\pm 2$

| -2 | | 2 |
|---|---|---|
| | 0 | |
| | − | |

top bottom

$$\int_{-2}^{2} 0-(x^2-4)=\int_{2}^{-2} x^2-4\,dx=\frac{x^3}{3}-4x\Big|_{2}^{-2}$$

changed limits

$$=\left(\frac{(-2)^3}{3}-4(-2)\right)-\left(\frac{2^3}{3}-4(2)\right)$$

$$=\left(-\frac{8}{3}+8\right)-\left(\frac{8}{3}-8\right)$$

$$\frac{16}{3}-\left(-\frac{16}{3}\right)=\frac{32}{3}$$

3) $e^{x/2}=0$

$\ln e^{x/2}=\cancel{\ln 0} \rightarrow$ DNE

No soln → no zeros

| 0 | | 4 |
|---|---|---|
| | 2 | |
| | + | |

top bottom

$$\int_{0}^{4} e^{x/2}-0\,dx=2e^{x/2}\Big|_{0}^{4}=2(e^2-e^0)$$

$$=2e^2-2$$

4) $\sin(3x)=0$ $\quad$ $x$: $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ $\quad$ $3x$: $\left[-\frac{3\pi}{2},\frac{3\pi}{2}\right]$

$3x=0$, $x=0$ $\quad$ $3x=\pi$, $x=\frac{\pi}{3}$ $\quad$ $3x=2\pi$ outside of domain $\quad$ $3x=-\pi$, $x=-\frac{\pi}{3}$ $\quad$ $3x=-2\pi$ outside of domain

| $-\frac{\pi}{2}$ | | $-\frac{\pi}{3}$ | | 0 | | $\frac{\pi}{3}$ | | $\frac{\pi}{2}$ |
|---|---|---|---|---|---|---|---|---|
| | $-\frac{5\pi}{12}$ | | $-\frac{\pi}{6}$ | | $\frac{\pi}{6}$ | | $\frac{5\pi}{12}$ | |
| | + | | − | | + | | − | |

$$\int_{-\frac{\pi}{2}}^{-\frac{\pi}{3}} \sin(3x)-0\,dx+\int_{-\frac{\pi}{3}}^{0} 0-\sin(3x)\,dx+\int_{0}^{\frac{\pi}{3}} \sin(3x)-0\,dx$$

$$+\int^{\frac{\pi}{2}} 0-\sin(3x)\,dx$$

$$\int_{-\frac{\pi}{2}}^{-\frac{\pi}{3}} \sin 3x\,dx + \int_{0}^{-\frac{\pi}{3}} \sin 3x\,dx + \int_{0}^{\frac{\pi}{3}} \sin 3x\,dx + \int_{\frac{\pi}{2}}^{\frac{\pi}{3}} \sin 3x\,dx$$

$$-\frac{\cos 3x}{3}\Bigg|_{-\frac{\pi}{2}}^{-\frac{\pi}{3}}$$

$$= \frac{\cos 3x}{3}\Bigg|_{-\frac{\pi}{3}}^{-\frac{\pi}{2}}$$

$$\frac{1}{3}\left(\cos\left(-\frac{3\pi}{2}\right) - \cos(-\pi)\right)$$

$$\frac{1}{3}\left(0 - (-1)\right)$$

$$\frac{1}{3}$$

$$-\frac{\cos 3x}{3}\Bigg|_{0}^{-\frac{\pi}{3}}$$

$$= \frac{\cos 3x}{3}\Bigg|_{-\frac{\pi}{3}}^{0}$$

$$\frac{1}{3}\left(\cos(0) - \cos(-\pi)\right)$$

$$\frac{1}{3}\left(1 - (-1)\right)$$

$$\frac{2}{3}$$

Do the same with the last 2 integrals

$$\frac{1}{3} + \frac{2}{3} + \frac{2}{3} + \frac{1}{3} = 2$$

5) $$\frac{\ln x^3}{x} = 0$$

$$\int_{\frac{1}{e}}^{1} 0 - \frac{\ln x^3}{x}\,dx + \int_{1}^{e^2} \frac{\ln x^3}{x} - 0\,dx = \int_{1}^{\frac{1}{e}} \frac{\ln x^3}{x}\,dx + \int_{1}^{e^2} \frac{\ln x^3}{x}\,dx$$

$\ln x^3 = 0$

$e^0 = x^3$

$1 = x^3$

$x = 1$

| $\frac{1}{e}$ | $1$ | $e^2$ |
|---|---|---|
| $\frac{2}{3}$ | $2$ | |
| $-$ | $+$ | |

$u = \ln x^3$

$du = \frac{1}{x^3}(3x^2)\,dx$

$dx = \frac{x}{3}\,du$

$$\int \frac{u}{\cancel{x}}\left(\frac{\cancel{x}}{3}\right)du = \frac{1}{3}\int u\,du = \frac{1}{3}\frac{u^2}{2} = \frac{1}{6}u^2$$

$$\frac{1}{6}u^2\Bigg|_{0}^{-3} = \frac{1}{6}\left((-3)^2 - 0^2\right) = \frac{3}{2}$$

limits of integration

$u = \ln\left(\frac{1}{e}\right)^3 = \ln(e^{-1})^3 = \ln e^{-3} = -3$

$u = \ln 1^3 = \ln 1 = 0$

$u = \ln(e^2)^3 = \ln e^6 = 6$

$$\frac{1}{6}u^2\Bigg|_{0}^{6} = \frac{1}{6}\left(6^2 - 0^2\right) = 6$$

$$6 + \frac{3}{2} = \frac{15}{2}$$

6) A) $x(x^2-1)=0$

$x=0 \quad x=\pm 1$ 1 is the only number between 0 and 2.

| 0 | | 1 | | 2 |
|---|---|---|---|---|
| | $\frac{1}{2}$ | | $\frac{3}{2}$ | |
| | $-$ | | $+$ | |

$$\int_0^1 0-(x(x^2-1))\,dx + \int_1^2 x(x^2-1)-0\,dx$$

$$\int_1^0 x^3-x\,dx + \int_1^2 x^3-x\,dx$$

$$= \frac{x^4}{4}-\frac{x^2}{2}\Big|_1^0 + \frac{x^4}{4}-\frac{x^2}{2}\Big|_1^2$$

$0-\left(\frac{1}{4}-\frac{1}{2}\right)$ $\qquad$ $\left(\frac{2^4}{4}-\frac{2^2}{4}\right)-\left(\frac{1}{4}-\frac{1}{2}\right)$

$\frac{1}{4}$ $\qquad$ $(4-2)-\left(-\frac{1}{4}\right)$

$\frac{9}{4}$

$$\frac{1}{4}+\frac{9}{4}=\frac{10}{4}=\frac{5}{2}$$

B) Negative area = Positive area

$\frac{1}{4}$ = positive area

from part A) $\quad \frac{1}{4}=\int_1^a x^3-x-0\,dx$

$$\frac{1}{4}=\frac{x^4}{4}-\frac{x^2}{2}\Big|_1^a$$

$$\frac{1}{4}=\left(\frac{a^4}{4}-\frac{a^2}{2}\right)-\left(\frac{1}{4}-\frac{1}{2}\right)$$

$$\frac{1}{4}=\left(\frac{a^4}{4}-\frac{a^2}{2}\right)+\frac{1}{4}$$

$$4(0)=\left(\frac{a^4}{4}-\frac{a^2}{2}\right)4$$

$$0=a^4-2a^2$$

$$0=a^2(a^2-2)$$

$a^2=0 \qquad a^2-2=0$

$a=0 \qquad a=\pm\sqrt{2}$

from picture, $a>1$

$\therefore a=\sqrt{2}$

7) $f(x) = x^{\ln x}$

$y = x^{\ln x}$

derivative

$\ln y = \ln x^{\ln x} = \ln x \cdot \ln x$

$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x}(\ln x) + \left(\frac{1}{x}\right)(\ln x)$

$\frac{1}{y}\frac{dy}{dx} = \frac{2\ln x}{x}$

$\frac{dy}{dx} = \frac{2\ln x}{x}(y) = \boxed{\frac{\ln x^2}{x}\left(x^{\ln x}\right)}$ ← more than 1 correct form for this answer

8) $\int \cos^3 x \, dx = \int \cos^2 x \cdot \cos x \, dx = \int (1-\sin^2 x)\cos x \, dx$

$u = \sin x$

$du = \cos x \, dx$

$dx = \frac{du}{\cos x}$

$= \int (1-u^2)(\cancel{\cos x})\left(\frac{du}{\cancel{\cos x}}\right) = \int 1-u^2 \, du$

$= u - \frac{u^3}{3} + C$

$= \boxed{\sin x - \frac{\sin^3 x}{3} + C}$

9) $f(x) = \int f'(x)\,dx = \int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

$f\left(\frac{1}{2}\right) = -\frac{5\pi}{6}$

$-\frac{5\pi}{6} = \sin^{-1}\left(\frac{1}{2}\right) + C$

$-\frac{5\pi}{6} = \frac{\pi}{6} + C$

$C = -\pi$

$\boxed{\therefore f(x) = \sin^{-1} x - \pi}$

10) $y = \frac{1}{12}x^4 - 2x^2$

$\frac{dy}{dx} = \frac{1}{3}x^3 - 4x$

$\frac{d^2y}{dx^2} = x^2 - 4 = 0$

$x = \pm 2$

| | -2 | | 2 | |
|---|---|---|---|---|
| -5 | | 0 | | 5 |
| + | | − | | + |

$\therefore$ concave up $(-\infty, -2), (2, +\infty)$

concave down $(-2, 2)$

inflection pt.

@ $(-2, -\frac{20}{3})$

$(2, \frac{?}{})$