# Area of a Bounded Region WS

1) $y = \frac{3x^2}{2} - 3$ , $y = \frac{x}{2} + 2$ (Area between 2 curves)

intersection:

$$\frac{3x^2}{2} - 3 = \frac{x}{2} + 2$$

$$3x^2 - 6 = x + 4$$

$$3x^2 - x - 10 = 0$$

$$(3x+5)(x-2) = 0$$

$$3x+5 = 0 \qquad x - 2 = 0$$

$$x = -\frac{5}{3} \qquad x = 2$$

$-\frac{5}{3}$ | 0 | 2

$y = \frac{3(0)^2}{2} - 3 = -3$

$y = \frac{0}{2} + 2 = 2$

$$\int_{-\frac{5}{3}}^{2} \frac{x}{2} + 2 - \left(\frac{3x^2}{2} - 3\right) dx$$

$$\int_{-5/3}^{2} -\frac{3x^2}{2} + \frac{x}{2} + 5 \, dx$$

$$-\frac{x^3}{2} + \frac{x^2}{4} + 5x \Big|_{-\frac{5}{3}}^{2}$$

$$\left(-\frac{2^3}{2} + \frac{2^2}{4} + 5(2)\right) - \left(-\frac{\left(-\frac{5}{3}\right)^3}{2} + \frac{\left(-\frac{5}{3}\right)^2}{4} + 5\left(-\frac{5}{3}\right)\right)$$

$$(-4 + 1 + 10) - \left(\frac{125}{54} + \frac{25}{36} - \frac{25}{3}\right) = 7 - \left(\frac{500}{216} + \frac{150}{216} - \frac{1800}{216}\right)$$

$$7 - \left(\frac{650}{216} - \frac{1800}{216}\right) = \frac{1512}{216} - \left(-\frac{1150}{216}\right) = \frac{2662}{216} = \frac{1331}{108}$$

2) $y = 5 - x$, $x = 4$, x-axis $(y=0)$, y-axis $(x=0)$ (Area under curve)

Zeros: $5 - x = 0$

$x = 5$ ← not in domain of $[0,4]$ so don't use

0 | 1 | 4

$y = 0$

$y = 5 - 1 = 4$

$$\int_0^4 5 - x - 0 \, dx = 5x - \frac{x^2}{2} \Big|_0^4 = \left(20 - \frac{4^2}{2}\right) - (0)$$

$$= 20 - 8 = 12$$

3) $y = x^2 - 4x - 5$, $y = 2x - 5$, $x = -2$, $x = 2$ (Area between 2 curves)

intersection: $x^2 - 4x - 5 = 2x - 5$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0$, $x = 6$

not in domain of $[-2, 2]$
so don't use

split integral up at 0.

| -2 | -1 | 0 | 1 | 2 |
|---|---|---|---|---|
| | $y = (-1)^2 - 4(-1) - 5 = 0$ | | $y = (1)^2 - 4(1) - 5 = -8$ | |
| | $y = 2(-1) - 5 = -7$ | | $y = 2(1) - 5 = -3$ | |

$$\int_{-2}^{0} (x^2 - 4x - 5) - (2x - 5)\,dx + \int_{0}^{2} (2x-5) - (x^2 - 4x - 5)\,dx$$

top bottom top bottom

$$\int_{-2}^{0} x^2 - 6x\,dx + \int_{0}^{2} -x^2 + 6x\,dx = \frac{44}{3} + \frac{28}{3} = 24$$

$$\frac{x^3}{3} - 3x^2 \Big|_{-2}^{0} \qquad -\frac{x^3}{3} + 3x^2 \Big|_{0}^{2}$$

$$0 - \left(\frac{(-2)^3}{3} - 3(-2)^2\right) \qquad \left(-\frac{2^3}{3} + 3(2)^2\right) - 0$$

$$0 - \left(-\frac{8}{3} - 12\right) \qquad -\frac{8}{3} + 12$$

$$\frac{44}{3} \qquad \frac{28}{3}$$

4) $y = x^3$, x-axis, $x = -1$ and $x = 2$ (Area under the curve)

Zeros: $x^3 = 0$

$x = 0$

split up integral

| -1 | $-\frac{1}{2}$ | 0 | 1 | 2 |
|---|---|---|---|---|
| | $y = 0$ | | $y = 0$ | |
| | $y = (-\frac{1}{2})^3 = -\frac{1}{8}$ | | $y = 1^3 = 1$ | |

$$\int_{-1}^{0} 0 - x^3\,dx + \int_{0}^{2} x^3 - 0\,dx = \int_{0}^{-1} x^3\,dx + \int_{0}^{2} x^3\,dx = \frac{1}{4} + 4 = \boxed{\frac{17}{4}}$$

$$\frac{x^4}{4}\Big|_{0}^{-1} \qquad \frac{x^4}{4}\Big|_{0}^{2}$$

$$\frac{1}{4} \qquad 4$$

5.)

$$f(x) = \begin{cases} \sin 2x, & x \le 0 \\ 3x, & x > 0 \end{cases}$$

$x = -\pi$, $x = 2$

$$\int_{-\pi}^{0} \sin 2x\,dx + \int_{0}^{2} 3x\,dx$$

continuous @ $x=0$?

$\sin 2(0) = \sin 0 = 0$

$3(0) = 0$

$\therefore$ continuous @ $x=0$

does $y = \sin 2x$ cross the x-axis between $-\pi$ and 0?

$\sin 2x = 0$

$2x = -\pi$ $\quad$ $2x = -2\pi$

$x = -\frac{\pi}{2}$ $\quad$ $x = -\pi$

only value in between limits of integration so split up integral at $-\frac{\pi}{2}$ and find top/bottom areas

does $y = 3x$ cross the x-axis between 0 and 2?

$3x = 0$

$x = 0$ Answer: No

$$= \int_{-\pi}^{-\frac{\pi}{2}} \underbrace{\sin(2x)}_{\text{top}} - \underbrace{0}_{\text{bottom}}\,dx + \int_{-\frac{\pi}{2}}^{0} \underbrace{0}_{\text{top}} - \underbrace{\sin(2x)}_{\text{bottom}}\,dx + \int_{0}^{2} \underbrace{3x}_{\text{top}} - \underbrace{0}_{\text{bottom}}\,dx$$

$$= \int_{-\pi}^{-\frac{\pi}{2}} \sin 2x\,dx - \int_{-\frac{\pi}{2}}^{0} \sin 2x\,dx + \int_{0}^{2} 3x\,dx = 1 + 1 + 6 = 8$$

$$-\frac{\cos 2x}{2}\Bigg|_{-\pi}^{-\frac{\pi}{2}} \qquad - \qquad -\frac{\cos 2x}{2}\Bigg|_{-\frac{\pi}{2}}^{0} \qquad \frac{3x^2}{2}\Bigg|_{0}^{2}$$

$$\frac{1}{2}\cos 2x\Bigg|_{-\frac{\pi}{2}}^{-\pi} \qquad \frac{1}{2}\cos 2x\Bigg|_{-\frac{\pi}{2}}^{0} \qquad \frac{3}{2}(4-0)$$

$$6$$

$$\frac{1}{2}\left(\cos(-2\pi) - \cos(-\pi)\right) \qquad \frac{1}{2}\left(\cos 0 - \cos(-\pi)\right)$$

$$\frac{1}{2}\left(1-(-1)\right) \qquad \frac{1}{2}\left(1-(-1)\right)$$

$$1 \qquad 1$$

6) $x = y^2$ and $y = x - 2$

$x = y + 2$

*intersection:*

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2 \quad y = -1$

$$\int_{-1}^{2} \overset{\text{Right}}{\overbrace{y+2}} - \overset{\text{Left}}{\overbrace{y^2}}\, dy$$

$$\frac{y^2}{2} + 2y - \frac{y^3}{3}\Big|_{-1}^{2} = \left(2 + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right)$$

$$= 6 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}$$

$$= 8 - \frac{9}{3} - \frac{1}{2} = 8 - 3 - \frac{1}{2}$$

$$= \frac{9}{2}$$

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7) $x = y^2$ and $x = 3 - 2y^2$

*intersection:*

$y^2 = 3 - 2y^2$

$3y^2 = 3$

$y^2 = 1$

$y = \pm 1$

$$\int_{-1}^{1} \overset{\text{Right}}{\overbrace{3-2y^2}} - \overset{\text{Left}}{\overbrace{y^2}}\, dy = \int_{-1}^{1} 3 - 3y^2\, dy$$

$$3y - y^3\Big|_{-1}^{1} = (3-1) - (-3-(-1))$$

$$= 2 - (-2) = 4$$

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8) $x = y^3 - y^2$ and $y = \frac{x}{2}$

$x = 2y$

*intersection*

$y^3 - y^2 = 2y$

$y^3 - y^2 - 2y = 0$

$y(y^2 - y - 2) = 0$

$y(y-2)(y+1) = 0$

$y = 0,\ y = 2,\ y = -1$

$$\int_{-1}^{0} \overset{\text{Right}}{\overbrace{y^3 - y^2}} - \overset{\text{Left}}{\overbrace{2y}}\, dy + \int_{0}^{2} \overset{\text{Right}}{\overbrace{2y}} - \overset{\text{Left}}{\overbrace{(y^3 - y^2)}}\, dy$$

$$\frac{y^4}{4} - \frac{y^3}{3} - y^2\Big|_{-1}^{0} \qquad y^2 - \frac{y^4}{4} + \frac{y^3}{3}\Big|_{0}^{2}$$

$$0 - \left(\frac{1}{4} + \frac{1}{3} - 1\right) \qquad \left(4 - 4 + \frac{8}{3}\right) - 0$$

$$\frac{5}{12} \quad + \quad \frac{8}{3} = \frac{37}{12}$$