

Area of a Bounded Region WS

1) $y = \frac{3x^2}{2} - 3$, $y = \frac{x}{2} + 2$ (Area between 2 curves)

Intersection:

$$\frac{3x^2}{2} - 3 = \frac{x}{2} + 2$$

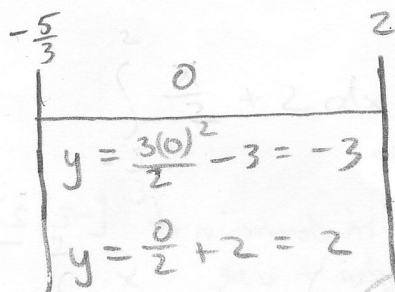
$$3x^2 - 6 = x + 4$$

$$3x^2 - x - 10 = 0$$

$$(3x+5)(x-2) = 0$$

$$3x+5=0 \quad x-2=0$$

$$x = -\frac{5}{3} \quad x = 2$$



$$\int_{-5/3}^2 \left(\frac{x}{2} + 2 - \left(\frac{3x^2}{2} - 3 \right) \right) dx$$

$$\int_{-5/3}^2 \left(-\frac{3x^2}{2} + \frac{x}{2} + 5 \right) dx$$

$$\left. -\frac{x^3}{2} + \frac{x^2}{4} + 5x \right|_{-5/3}^2$$

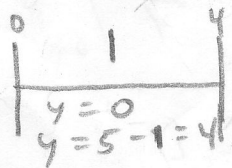
$$\left(-\frac{2^3}{2} + \frac{2^2}{4} + 5(2) \right) - \left(-\frac{(-5/3)^3}{2} + \frac{(-5/3)^2}{4} + 5\left(-\frac{5}{3}\right) \right)$$

$$\left(-4 + 1 + 10 \right) - \left(\frac{125}{54} + \frac{25}{36} - \frac{25}{3} \right) = 7 - \left(\frac{500}{216} + \frac{150}{216} - \frac{1800}{216} \right)$$

$$7 - \left(\frac{650}{216} - \frac{1800}{216} \right) = \frac{1512}{216} - \left(-\frac{1150}{216} \right) = \frac{2662}{216} = \frac{1331}{108}$$

2) $y = 5 - x$, $x = 4$, x -axis ($y = 0$), y -axis ($x = 0$) (Area under curve)

Zeros: $5 - x = 0$
 $x = 5 \notin [0, 4]$ not in domain of $[0, 4]$ so don't use



$$\int_0^4 (5 - x - 0) dx = 5x - \frac{x^2}{2} \Big|_0^4 = \left(20 - \frac{4^2}{2} \right) - (0)$$

$$= 5(4) - \frac{4}{2} = 20 - 8 = 12$$

3) $y = x^2 - 4x - 5$, $y = 2x - 5$, $x = -2$, $x = 2$ (Area between 2 curves)

Intersection: $x^2 - 4x - 5 = 2x - 5$

$x^2 - 6x = 0$

$x(x-6) = 0$

$x = 0$, $x = 6$

↑
not in domain of $[-2, 2]$
So don't use

split integral up at 0.

-2	-1	0	1	2
$y = (-1)^2 - 4(-1) - 5$		$y = (1)^2 - 4(1) - 5$		
$= 0$		$= -8$		
$y = 2(-1) - 5$		$y = 2(1) - 5$		
$= -7$		$= -3$		

$\int_{-2}^0 (x^2 - 4x - 5) - (2x - 5) dx + \int_0^2 (2x - 5) - (x^2 - 4x - 5) dx$

↑ ↑ ↑ ↑
top bottom top bottom

$\int_{-2}^0 x^2 - 6x dx + \int_0^2 -x^2 + 6x dx = \frac{44}{3} + \frac{28}{3} = 24$

$\left. \frac{x^3}{3} - 3x^2 \right|_{-2}^0$

$\left. -\frac{x^3}{3} + 3x^2 \right|_0^2$

$0 - \left(\frac{(-2)^3}{3} - 3(-2)^2 \right)$

$\left(-\frac{2^3}{3} + 3(2)^2 \right) - 0$

$0 - \left(-\frac{8}{3} - 12 \right)$

$-\frac{8}{3} + 12$

$\frac{44}{3}$

$\frac{28}{3}$

4) $y = x^3$, x-axis, $x = -1$ and $x = 2$ (Area under the curve)

Zeros:

$x^3 = 0$

$x = 0$

↑
split up integral

$\int_{-1}^0 0 - x^3 dx + \int_0^2 x^3 - 0 dx = \int_0^{-1} x^3 dx + \int_0^2 x^3 dx = \frac{1}{4} + 4 = \frac{17}{4}$

$\left. \frac{x^4}{4} \right|_0^{-1}$

$\left. \frac{x^4}{4} \right|_0^2$

$\frac{1}{4}$

4

-1	-1/2	0	1	2
$y = 0$		$y = 0$		
$y = (-1/2)^3 = -1/8$		$y = 1^3 = 1$		

5.)

$$f(x) = \begin{cases} \sin 2x, & x \leq 0 \\ 3x, & x > 0 \end{cases}$$

$$x = -\pi, x = 2$$

continuous @ $x=0$?

$$\sin 2(0) = \sin 0 = 0$$

$$3(0) = 0$$

\therefore continuous @ $x=0$

$$\int_{-\pi}^0 \sin 2x dx + \int_0^2 3x dx$$

does $y = \sin 2x$ cross the x-axis between $-\pi$ and 0?

$$\sin 2x = 0$$

$$2x = -\pi \quad 2x = -2\pi$$

$$x = -\frac{\pi}{2} \quad x = -\pi$$

↑

only value in between

limits of integration so split up integral at $-\frac{\pi}{2}$ and find top/bottom areas

does $y = 3x$ cross the x-axis between 0 and 2?

$$3x = 0$$

$x = 0$ Answer: No

$$= \int_{-\pi}^{-\frac{\pi}{2}} \sin(2x) - 0 dx + \int_{-\frac{\pi}{2}}^0 0 - \sin(2x) dx + \int_0^2 3x - 0 dx$$

↑ top
↑ bottom
↑ top
↑ bottom
↑ top
↑ bottom

$$= \int_{-\pi}^{-\frac{\pi}{2}} \sin 2x dx - \int_{-\frac{\pi}{2}}^0 \sin 2x dx + \int_0^2 3x dx = 1 + 1 + 6 = 8$$

$$= \left. \frac{-\cos 2x}{2} \right|_{-\pi}^{-\frac{\pi}{2}} - \left. \frac{-\cos 2x}{2} \right|_{-\frac{\pi}{2}}^0 + \left. \frac{3x^2}{2} \right|_0^2$$

$$\frac{1}{2} \cos 2x \Big|_{-\pi}^{-\frac{\pi}{2}} - \frac{1}{2} \cos 2x \Big|_{-\frac{\pi}{2}}^0 + 6$$

$$\frac{1}{2} (\cos(-2\pi) - \cos(-\pi)) - \frac{1}{2} (\cos 0 - \cos(-\pi))$$

$$\frac{1}{2} (1 - (-1)) - \frac{1}{2} (1 - (-1))$$

1

1

6) $x = y^2$ and $y = x - 2$

$x = y + 2$

Intersection:

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y - 2)(y + 1) = 0$

$y = 2 \quad y = -1$

Right Left
 \downarrow \downarrow
 $\int_{-1}^2 y + 2 - y^2 dy$

$\left. \frac{y^2}{2} + 2y - \frac{y^3}{3} \right|_{-1}^2 = \left(2 + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} - 2 + \frac{1}{3} \right)$

$= 6 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}$

$= 8 - \frac{9}{3} - \frac{1}{2} = 8 - 3 - \frac{1}{2}$

$= \boxed{\frac{9}{2}}$

7) $x = y^2$ and $x = 3 - 2y^2$

Intersection:

$y^2 = 3 - 2y^2$

$3y^2 = 3$

$y^2 = 1$

$y = \pm 1$

Right Left
 \downarrow \downarrow
 $\int_{-1}^1 3 - 2y^2 - y^2 dy = \int_{-1}^1 3 - 3y^2 dy$

$\left. 3y - y^3 \right|_{-1}^1 = (3 - 1) - (-3 - (-1))$

$= 2 - (-2) = \boxed{4}$

8) $x = y^3 - y^2$ and $y = \frac{x}{2}$

$x = 2y$

Intersection:

$y^3 - y^2 = 2y$

$y^3 - y^2 - 2y = 0$

$y(y^2 - y - 2) = 0$

$y(y - 2)(y + 1) = 0$

$y = 0, y = 2, y = -1$

Right Left Right Left
 \downarrow \downarrow \downarrow \downarrow
 $\int_{-1}^0 y^3 - y^2 - 2y dy + \int_0^2 2y - (y^3 - y^2) dy$

$\left. \frac{y^4}{4} - \frac{y^3}{3} - y^2 \right|_{-1}^0$

$0 - \left(\frac{1}{4} + \frac{1}{3} - 1 \right)$

$\frac{5}{12}$

$\left. y^2 - \frac{y^4}{4} + \frac{y^3}{3} \right|_0^2$

$\left(4 - 4 + \frac{8}{3} \right) - 0$

$\frac{8}{3}$

$= \boxed{\frac{37}{12}}$